## Questions

**Example (2.6.1)** Determine if the differential equation $(2x + 3) + (2y - 2)y' = 0$ is exact. If it is exact, find the solution.

**Example (2.6.16)** Find the value of $b$ which makes the differential equation $(ye^{2xy} + x)dx + bxe^{2xy}dy = 0$ exact, then solve the equation for that value of $b$.

## Solutions

**Example (2.6.1)** Determine if the differential equation $(2x + 3) + (2y - 2)y' = 0$ is exact. If it is exact, find the solution.

The standard form for determining if an equation is exact is $M + Ny' = 0$.

Identify $M$ and $N$:

$$M(x, y) = 2x + 3, \quad N(x, y) = 2y - 2$$

$$M_y(x, y) = 0 = N_x(x, y)$$

So the DE is exact. Therefore, we look for a $\psi$ that satisfies:

$$\psi_x(x, y) = M(x, y) = 2x + 3$$

$$\psi_y(x, y) = N(x, y) = 2y - 2$$

Let's integrate the second equation, to get:

$$\psi(x, y) = \int N(x, y)dy = y^2 - 2y + h(x)$$

Now, we differentiate with respect to $x$:

$$\psi_x(x, y) = M(x, y) = \frac{dh(x)}{dx}$$

Now, we compare the two equations for $M(x, y)$, and identify that $h(x)$ must solve the equation:

$$\frac{dh(x)}{dx} = 2x + 3$$

$$h(x) = x^2 + 3x + C = x^2 + 3x$$

We do not include any constants of integration here, much like when we dropped the constants of integration when we solved the DE for the integrating factor. The constants are all collected in the final solution

And so the complete solution is given implicitly by:

$$\psi(x, y) = C \rightarrow y^2 - 2y + x^2 + 3x = C.$$

**Example (2.6.16)** Find the value of $b$ which makes the differential equation $(ye^{2xy} + x)dx + bxe^{2xy}dy = 0$ exact, then solve the equation for that value of $b$.

The standard form for determining if an equation is exact is $M + Ny' = 0$.

Identify $M$ and $N$:

$$M(x, y) = ye^{2xy} + x, \quad N(x, y) = bxe^{2xy}$$

$$M_y(x, y) = e^{2xy} + 2xye^{2xy}$$

$$N_x(x, y) = be^{2xy} + 2bxye^{2xy}$$

The DE will be exact if $b = 1$.

Therefore, we look for a $\psi$ that satisfies:

$$\psi_x(x,y) = M(x,y) = ye^{2xy} + x$$

$$\psi_y(x,y) = N(x,y) = xe^{2xy}$$

Let's integrate the second equation, to get:

$$\psi(x,y) = \int N(x,y)dy = \int xe^{2xy}dy = x\int e^{2xy}dy = \frac{1}{2}e^{2xy} + h(x)$$

Now, we differentiate with respect to $x$:

$$\psi_x(x,y) = M(x,y) = ye^{2xy}\frac{dh(x)}{dx}$$

Now, we compare the two equations for $M(x,y)$, and identify that $h(x)$ must solve the equation:

$$\frac{dh(x)}{dx} = x$$

$$h(x) = \frac{x^2}{2} + C = \frac{x^2}{2}$$

We do not include any constants of integration here, much like when we dropped the constants of integration when we solved the DE for the integrating factor. The constants are all collected in the final solution

And so the complete solution is given implicitly by:

$$\psi(x,y) = C \rightarrow \frac{1}{2}e^{2xy} + \frac{x^2}{2} = C.$$

In this case we can solve for an explicit solution,

$$y(x) = \frac{\ln(2C - x^2)}{2x}.$$