

## Questions

**Example (6.3.4)** Sketch  $g(t) = f(t-3)u_3(t)$ , where  $f(t) = \sin(t), t \geq 0$ .

**Example (6.3.7)** Find the Laplace transform of  $f(t) = \begin{cases} 0, & t < 2 \\ (t-2)^2, & t \geq 2 \end{cases}$ .

**Example (6.3.18)** Find the inverse Laplace transform of  $F(s) = \frac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}$ .

## Solutions

**Example (6.3.4)** Sketch  $g(t) = f(t-3)u_3(t)$ , where  $f(t) = \sin(t), t \geq 0$ . See *Mathematica* notebook for sketch.

**Example (6.3.7)** Find the Laplace transform of  $f(t) = \begin{cases} 0, & t < 2 \\ (t-2)^2, & t \geq 2 \end{cases}$ .

We want to write  $f(t)$  in terms of  $u_c(t)$  so we can use our table of Laplace transforms.

$$f(t) = u_2(t)(t-2)^2.$$

$$\begin{aligned} \mathcal{L}[f(t)] &= \int_0^{\infty} e^{-st} f(t) dt \\ &= \int_0^{\infty} e^{-st} u_2(t)(t-2)^2 dt \\ &= \int_2^{\infty} e^{-st} (t-2)^2 dt \end{aligned}$$

use *Mathematica* to do the integral, or use parts

$$= \frac{2e^{-2s}}{s^3}, \quad s > 0$$

**Example (6.3.18)** Find the inverse Laplace transform of  $F(s) = \frac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}$ .

Here we just need to use Table 6.2.1 #12 repeatedly.

$$\begin{aligned} \mathcal{L}[f(t)] &= \mathcal{L}^{-1}\left[\frac{e^{-s}}{s}\right] + \mathcal{L}^{-1}\left[\frac{e^{-2s}}{s}\right] - \mathcal{L}^{-1}\left[\frac{e^{-3s}}{s}\right] - \mathcal{L}^{-1}\left[\frac{e^{-4s}}{s}\right] \\ &= u_1(t) + u_2(t) - u_3(t) - u_4(t) \end{aligned}$$