Virtually all these questions come from some edition of Boyce and DiPrima. I have left the problem number on the questions, but they may not line up with the current edition of the textbook.

Questions

Determine the order of the given differential equation, also state whether the equation is linear or nonlinear.

Example (1.3.1) $t^2\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + 2y = \sin t$

Example (1.3.2) $(1+y^2)\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + y = e^t$

Example (1.3.3) $\dfrac{d^4y}{dt^4} + \dfrac{d^3y}{dt^3} + \dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + y = 1$

Example (1.3.4) $\dfrac{dy}{dt} + ty^2 = 0$

Example (1.3.5) $\dfrac{d^2y}{dt^2} + \sin(t+y) = \sin t$

Example (1.3.6) $\dfrac{d^3y}{dt^3} + t\dfrac{dy}{dt} + (\cos^2 t)y = t^3$

Example (1.3.7) Verify that $y_1(t) = e^t$ and $y_2(t) = \cosh t$ are solutions of the differential equation $y'' - y = 0$.

Example (1.3.14) Verify that $y = e^{t^2}\displaystyle\int_0^t e^{-s^2}\,ds + e^{t^2}$ is a solution of the differential equation $y' - 2ty = 1$.

Solutions

Determine the order of the given differential equation, also state whether the equation is linear or nonlinear.

Example (1.3.1) $t^2\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + 2y = \sin t$

Second order; linear.

Example (1.3.2) $(1+y^2)\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + y = e^t$

Second order; nonlinear. This is nonlinear due to the $y^2\dfrac{d^2y}{dt^2}$ term.

Example (1.3.3) $\dfrac{d^4y}{dt^4} + \dfrac{d^3y}{dt^3} + \dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + y = 1$

Fourth order; linear.

Example (1.3.4) $\dfrac{dy}{dt} + ty^2 = 0$

First order; nonlinear. This is nonlinear due to the y^2.

Example (1.3.5) $\dfrac{d^2y}{dt^2} + \sin(t+y) = \sin t$

Second order; nonlinear. This is nonlinear due to the $\sin(t+y)$, which in nonlinear in y.

Example (1.3.6) $\dfrac{d^3y}{dt^3} + t\dfrac{dy}{dt} + (\cos^2 t)y = t^3$

Third order; linear.

Example (1.3.7) Verify that $y_1(t) = e^t$ and $y_2(t) = \cosh t$ are solutions of the differential equation $y'' - y = 0$.

To verify the solutions, we need to calculate derivatives and substitute into the differential equation.

$$\begin{aligned} y_1(t) &= e^t \\ y_1'(t) &= e^t \\ y_1''(t) &= e^t \end{aligned}$$

We have

$$y_1'' - y_1 = e^t - e^t = 0$$

Therefore, $y_1(t) = e^t$ satisfies the differential equation.

$$\begin{aligned} y_2(t) &= \cosh t \\ y_2'(t) &= \sinh t \\ y_2''(t) &= \cosh t \end{aligned}$$

We have

$$y_2'' - y_2 = \cosh t - \cosh t = 0$$

Therefore, $y_2(t) = \cosh t$ satisfies the differential equation.

Example (1.3.14) Verify that $y = e^{t^2} \int_0^t e^{-s^2}\, ds + e^{t^2}$ is a solution of the differential equation $y' - 2ty = 1$.

To verify the solution, we need to calculate derivatives and substitute into the differential equation.

$$
\begin{aligned}
y &= e^{t^2} \int_0^t e^{-s^2}\, ds + e^{t^2} \\
y' &= \frac{d}{dt}\left[e^{t^2} \int_0^t e^{-s^2}\, ds + e^{t^2}\right] \\
&= e^{t^2} \frac{d}{dt}\left[\int_0^t e^{-s^2}\, ds\right] + \frac{d}{dt}\left[e^{t^2}\right] \int_0^t e^{-s^2}\, ds + \frac{d}{dt}\left[e^{t^2}\right] \\
&= e^{t^2}\left[e^{-t^2}\right] + \left[2te^{t^2}\right] \int_0^t e^{-s^2}\, ds + \left[2te^{t^2}\right] \\
&= 1 + 2te^{t^2} \int_0^t e^{-s^2}\, ds + 2te^{t^2}
\end{aligned}
$$

We have

$$
y'' - 2ty = 1 + 2te^{t^2} \int_0^t e^{-s^2}\, ds + 2te^{t^2} - 2t\left(e^{t^2} \int_0^t e^{-s^2}\, ds + e^{t^2}\right) = 1
$$

Therefore, $y = e^{t^2} \int_0^t e^{-s^2}\, ds + e^{t^2}$ satisfies the differential equation.