Math 4401: NM Assignment 1 Due: Feb 12, 2008

Your solutions can contain *Mathematica* output and handwritten sheets. Don't try to spend too much time typesetting on *Mathematica*–but you should add enough details to make the *Mathematica* file understandable!

If your *Mathematica* file is long, supress unnecessary output and scale diagrams to reduce its size. If your *Mathematica* solution is 20 pages long, then talk to me before you print it out! I will probably want you to give me a shortened version on paper, and you can then email me the complete *Mathematica* file for my pleasure.

Remember–talk to me and your peers if you have any questions.

(20) 1. 1.1.10 from the text. This question involves Taylor series with error term.

(20) 2. 1.2.23 from the text.

The arctangent function can be expanded as a series, and used to approximate π. This problem presents two such ways of approximating π,

$$\pi = \lim_{n\to\infty} 4\left(P_n(1/2) + P_n(1/3)\right),$$

$$\pi = \lim_{n\to\infty} \left(16P_n(1/5) - 4P_n(1/239)\right).$$

We obviously want to use *Mathematica* instead of Maple for this problem. You can define the functions above symbolically, and then wrap a `SetPrecision` around them to get higher than 16 digits. Since the book suggests 75 digits of accuracy, you should set the precision to that.

I used the `MantissaExponent` and `Part` commands to get a nice table of the exponent that we are interested in for various values of n.

To define the Maclaurin series P_n, use the command `Derivative` and `Sum` to construct the series. This is the easiest way to go.

You could also use the `Series` and `Normal` commands, but make sure you don't substitute for x too early!

(20) 3. 2.2.10 from the text; does the fixed point theorem work if you use $x = g(x) = 25/x^2$? Why or why not?

Exercise 2.1.13 found the result using the bisection method accurate to 10^{-4} after 14 iterations with initial bounds of $[2, 3]$.

You may find the command `Nest` useful for this problem.

(20) 4. 2.3.17 from the text.

Output all the approximations in your procedures, $p_i, f(p_i)$.

(20) 5. 2.5.17 from the text.

See Hint in back of text for part a).