Math 4401: NM Assignment 2 Due: Feb 26, 2008

Your solutions can contain *Mathematica* output and handwritten sheets. Don't try to spend too much time typesetting on *Mathematica*–but you should add enough details to make the *Mathematica* file understandable!

If your *Mathematica* file is long, supress unnecessary output and scale diagrams to reduce its size. If your *Mathematica* solution is 20 pages long, then talk to me before you print it out! I will probably want you to give me a shortened version on paper, and you can then email me the complete *Mathematica* file for my pleasure.

Remember–talk to me and your peers if you have any questions.

(20) 1. Given the function $f(x) = x\ln(x+1)$, we have seen how to construct the Taylor polynomial approximation $T_{20}(x)$ and the Padé approximant $[L, M] = [10, 10]$.

Construct these approximations, and graph the errors $|f(x) - T_{20}(x)|$ and $|f(x) - [10, 10]|$.

The Padé approximant concept can be extended to algebraic approximants, which we can call $[L, M, N]$, in the following manner.

Polynomials $P_L(x)$, $Q_M(x)$, and $R_N(x)$ can be used to form the quantity

$$P_L(x) + T_n(x)Q_M(x) + (T_n(x))^2 R_N(x) = 0.$$

Proceeding as we did for the Padé approximants, you can collect powers of x, then set coefficients of the first $L + M + N + 2$ coefficents equal to zero to solve for the $L + M + N + 2$ unknowns (we can set one unknown equal to 1, so pick $q_0 = 1$).

The $[L, M, N]$ approximants are then found by the solving the equation

$$P_L(x) + [L, M, N]Q_M(x) + ([L, M, N])^2 R_N(x) = 0,$$

for $[L, M, N]$. Since this is a quadratic, it can be done by hand and you will find two algebraic approximants, $[L, M, N]_1$ and $[L, M, N]_2$.

Construct the $[5, 5, 5]_1$ and $[5, 5, 5]_2$ algebraic approximants, and graph the errors $|f(x)-[5, 5, 5]_1|$ and $|f(x)-[5, 5, 5]_1|$. Comment on the behaviour of the two algebraic approximants.

What benefit does the algebraic approximant provide us? One benefit is seen if you evaluate $f(-2)$, $T_{20}(-2)$, and the Padé and the algebraic approximants at $x = -2$. Comment on what you find.

(20) 2. Given $f(x) = x^3 + 2x^2 + 10x - 20$, find all three roots (complex and real valued) to 10^{-8} using Müller's method.

You may want to use a random search of phase space to help you find all the roots. Randomly sample phase space points and see what roots are found (you can wrap this in a `Do` loop if you like, and use the *Mathematica* command `Random`). If you do use this method, comment on how your random search worked–which root was most likely found? Was a root difficult to find?

(20) 3. 3.1.28 from the text. Include in your solution a sketch of the data points and the approximating polynomial you used.

(20) 4. 3.4.30 from the text.

You should plot the velocity as a function of time, and explain what characteristics of this plot let you know that the cubic spline is a free cubic spline.

(20) 5. 4.1.28 from the text.

You won't need *Mathematica* to do this problem. The goal is to write four equations, and then combine them in an ingenious way to solve for $f^{(3)}(x_0)$ in terms of $f(x_0 + h)$, $f(x_0 - h)$, $f(x_0 + 2h)$, $f(x_0 - 2h)$. The error term should just tag along, and when you are done you will see that the error term is $O(h^2)$.

I suggest you work with the pair $f(x_0 + h)$ and $f(x_0 - h)$ first, combining them to get a new equation. Then, work with $f(x_0 + 2h)$ and $f(x_0 - 2h)$, again combining them to get a new equation.