Math 4401: NM Assignment 3 Due: Mar 13, 2008

Your solutions can contain *Mathematica* output and handwritten sheets. Don't try to spend too much time typesetting on *Mathematica*–but you should add enough details to make the *Mathematica* file understandable! Email me your *Mathematica* file, and turn in any handwritten pages during class.

Remember–talk to me and your peers if you have any questions.

(20) 1. 4.1.26 from the text. Get at least two approximations to the voltage, using any two of the techniques we have learned in class.

My text has an error in the table, listing 1.04 as simply 1.0.

(20) 2. 4.4.26 from the text. Is it possible to get a rigorous error bound in this case?

(20) 3. Use Romberg integration to find R_{55} to approximate

$$\int_0^{\pi/2} e^{x^2} \sin x \, dx.$$

Then use Romberg integration that uses the Midpoint rule rather than the trapezoidal rule to calculate the integral.

Something to think about: Why should we not use Simpson's rule to "improve" Romberg integration? I'll provide an answer in the solutions.

(20) 4. The zeta function $\zeta(n)$, $n = 2, 3, 4, \ldots$ can be defined by a multidimensional integral,

$$\zeta(n) = \frac{2^{n-1}}{2^{n-1} - 1} \int_0^1 dx_1 \int_0^1 dx_2 \cdots \int_0^1 dx_n \frac{1}{1 + x_1 x_2 \cdots x_n}.$$

Using quasi-random sequences for a 5-dimensional space based on the Corput sequence, estimate the value of $\zeta(4)$ using quasi-Monte Carlo integration to evaluate the integral

$$\zeta(4) = \frac{8}{7} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^1 dx_4 \frac{1}{1 + x_1 x_2 x_3 x_4}.$$

Sketch the error and estimate of $\zeta(4)$, for every 500th value from your Monte Carlo process.

If you want to challenge yourself, set your Monte Carlo routine up so it will work for a general value of n. Then you can easily calculate estimate for $\zeta(n)$ for any value of n!

Note that in practice it is much easier to approximate the zeta function using $\zeta(n) = \sum_{i=1}^{\infty} \frac{1}{i^k}$.

(20) 5. 8.1.5 from the text, parts a, b and e only!