Math 4401: NM Assignment 4 Due: Apr 17, 2008

Your solutions can contain *Mathematica* output and handwritten sheets. Don't try to spend too much time typesetting on *Mathematica*–but you should add enough details to make the *Mathematica* file understandable! Email me your *Mathematica* file, and turn in any handwritten pages during class.

Remember–talk to me and your peers if you have any questions.

(10) 1. 5.1.1 part d from the text.

(10) 2. 5.2.12 from the text.

(20) 3. 5.4.29 from the text.

You should do a few runs, varying the step size, until you get an answer which is accurate to one decimal.

(20) 4. 5.6.13 from the text.

This is easiest written out by hand. You shouldn't try to typeset the solution to this problem!

(20) 5. 6.1.10 from the text.

You can use the *Mathematica* command `RowReduce` to solve the system of equations (this command performs Gaussian Elimination), or Cramer's rule.

Set $\alpha = 1.0001$ in the system of equations, and solve.

Set $\alpha = 0.9999$ in the system of equations, and solve.

Make some comments about the solutions you found, explaining anything surprising you notice.

Based on what you've seen above, if you use Cramer's Rule to solve a system of equations, can you think of a condition to look for that would alert you to the fact that the system is ill-conditioned?

(20) 6. Chua's Circuit is a dynamical system given by

$$\begin{aligned}
\frac{dx}{dt} &= \alpha(y - x - f(x)) \\
\frac{dy}{dt} &= x - y + z \\
\frac{dz}{dt} &= -\beta y \\
& x(0) = x_0, y(0) = y_0, z(0) = z_0
\end{aligned}$$

Modifying this slightly so $f(x) = f(x, y, z) = \dfrac{\cos(x^3 + y - z^2)}{1 + x^2 + y^4}$, and using the parameters $\alpha = 13$ and $\beta = 64$ gives us a nice little dynamical system to play with.

(a) Determine the solution out to time $t = 10$ accurately using Runga-Kutta for the initial starting point of $(0.1, 0.15, 0.05)$ (you could use something else if you want, but Runga-Kutta is really good at this). Plot the solution as a function of time using the *Mathematica* command `ListPointPlot3D`.

(b) Determine the solution out to time $t = 10$ accurately using Runga-Kutta for the initial starting point of $(0.16, 0.15, 0.05)$. Plot the solution as a function of time.

(c) How far apart are the two trajectories at time $t = 10$? In your opinion, is this system displaying a high degree of chaos?

If you want to learn a little more about Chua's Circuit, check out
http://www.cmp.caltech.edu/ mcc/chaos_new/Chua.html