Course Guide for Math 4452: Topics in Applied Math Mathematical Modeling (4 cr)

> *Some people are fluent in English, and some people are fluent in calculus–we have plenty of each. We need more people who are fluent in both languages and are willing and able to translate. These are the people who will be influential in solving the problems of the future.*
>
> –Mark Meerschaert, *Mathematical Modeling*, 2nd Ed. Academic Press, San Diego, 1999.

Session: Spring 2002
Time: TTh 12:00pm–1:40pm
Location: First Week: Social Science 130. **Starting Jan 22, Move to Sci 3655**
Instructor: Barry McQuarrie
Office: Science 1380
Office Hours: M 3:30–4:30pm; T 8:00–10:00am; W 1:00–2:00pm; F 12:00–1:00pm (or by appointment)
email: mcquarrb@mrs.umn.edu
My spring 2002 schedule: http://cda.mrs.umn.edu/∼mcquarrb/schedule.html
Webpage: http://cda.mrs.umn.edu/∼mcquarrb/
Course webpage: http://cda.mrs.umn.edu/ mcquarrb/Modeling/MM.html

Goals

Upon completion of this course a student should be able to:

- Demonstrate understanding of the mathematical modeling process.
- Apply the mathematical modeling process to a variety of situations from the real world.
- Know the strengths and limitations of mathematical modeling as a method for solving real-world problems.
- Exhibit competence with the use of technology in the modeling process.
- Use a variety of mathematical techniques in modeling and problem solving.
- Model from data.
- Model from theory.
- Communicate the results of a model effectively, both orally and in writing.

Textbook

Mathematical Modeling 2nd Ed. by Mark M. Meerschaert.

There is a **webpage** for the text: http://unr.edu/homepage/mcubed/modeling.html. There is a list of typos that made their way into the second edition, so you may want to visit the site and make the corrections to your text.

Mathematica

Mathematica has many useful features to someone interested in modeling. It can produce lovely plots of data sets and model fits, which can be an excellent way to present your model. It also has linear and nonlinear fitting packages to help you pick a model which best reproduces the data. And since we rarely choose the correct model to begin, the process of refining your model is helped by *Mathematica*. *Mathematica* also has text editing capabilities, so an entire report can be created directly in *Mathematica*.

I have quite a few *Mathematica* notebooks on my web page from the various courses I have taught over the years. These may serve as a useful guide. On my web page I will be placing *Mathematica* notebooks for you to download and experiment with. As well, there is an abundance of *Mathematica* assistance available on the web, and you should use those resources as you see fit. My web page has links to some relevant sites, and the UMM math department web page has links as well.

The focus of the course is the study of different methods of modeling physical systems, and the communication of those results to others. As such, we will not be too concerned with the numerical methods that will be used (although we will talk about them occasionally). If we can use *Mathematica* to solve a system of equations, a differential equation, or to fit curves to data, we shall. Numerical methods are studied in the course MATH 4401 Numerical Methods (which, by the way, I shall be teaching in Spring of 2003).

Grading

There will be no tests in this class. Instead, you will be graded on class participation, homework, and a final project. The course marks will be split in the following fashion:

Homework	60%
Participation	20%
Final Project	20%

Homework 60%

There will be a selection of homework questions assigned throughout the semester. Since an important aspect of the course is the communication of ideas, you should concentrate on first solving the problem, and then communicating the solution in a manner understandable to others.

I will also be asking you to describe in detail the process that went into arriving at the solution.

No late homework will be accepted unless you have made arrangements with me prior to the due date of the homework.

Participation 20%

During class we will be discussing concepts and ideas and the importance of how we present a solution to others. As such, class time will be spent in discussion and lecturing. The important aspects of your participation during a class session are:

- attendance,
- preparation (reading the relevant sections before class),
- weekly journal submissions,
- ongoing discussion with your classmates and me.

Journal Submissions I would like each of you to email me a short *journal entry* each Monday by noon based on the assigned reading for the coming week. This is an important aspect of the class, since it will help me guide the focus of the week's lectures and help create a continuing dialogue between you and me. This is especially important in a small class where students may have a wide variety of background experience! For example, if half the class has not seen Lagrange Multipliers, I certainly need to spend more time in class on it than if everyone has encountered them before.

The journal entry should include a discussion of the important ideas from the reading, whether you have seen these ideas before (and if you have, in what context) and any questions you may have. They may also contain discussion on the progress of your learning in the course, and strategies that you are going to help you learn the things you want to know.

As time progresses, your journal entries may contain reflections about a technique which you are having difficulty understanding, the possible limitations of a technique we have learned, or further applications of the technique to other problems. I am not looking for *correct* reflection, or specific examples–I simply want to see what sorts of reflections you have had over the week. I will be providing you with my own reflections each week.

To help you get started, I am included my journal entry for Chapter One. As you can see, you should feel free to let your personality shine through!

Barry's Journal Entry for Week 1 Chapter One of the text introduces (on the very first page!) the Five–Step Method to create mathematical models. It reminds me of when professors used to say "Write a sentence that answers the question". I guess they wanted me to show my thought process, but they never said that–the phrase "sentence that answers the question" was the only clue I had that my explanations could (and should!) go beyond mathematical symbols.

The Five–Step Method is used to solve a problem about selling a pig. I like that a very simple problem is used to help me understand the method (single variable optimization, which I first saw as a freshman)–I can focus on the presentation rather than on the mathematics being used. I am sure that the mathematics will become more cumbersome soon enough!

In differential equations, we could (and sometimes I even did!) check our answer to see if it was right when we were done. So we knew that the function we found was the correct solution. But we never asked if the question that had been asked was the correct question! Sensitivity analysis seems to be what is needed to check if the question is the correct question (or should I say "the model is correctly predicting reality"?). And better yet–it can tell you that it is the wrong question but "correct enough" to give reasonable , realistic results. Sensitivity analysis answers questions about your assumptions...and those are questions I have not spent a lot of time thinking about. I can see that that is going to change!

Robustness is the result of sensitivity analysis. If the model is not sensitive to changes in the parameters, and models reality fairly well, we can say the model is robust. However, if small changes in the model result in large changes in the type of behaviour of the solution, we would say the model is not very robust. I wrote this out because I remember a prof talking about robustness and I didn't really understand what he meant.

In creating mathematical models, we need to be aware of sensitivity, robustness, and how well we communicate our results.

The Final Project 20%

The final project will serve as a capstone to the course, and replace the traditional final exam. The goal of the final project is to work constructively in a group setting toward a common goal, drawing on the strengths of the individuals in the group, and to present your work to the class via a presentation, and to me via a written report (8–15 pages). Since there are eight students in the class, I believe that working in groups of two will be the best way to proceed.

The final project should be a significant expansion of a model considered during the class, or a new model which we have not covered at all in class. Most importantly, it should be a topic which is of interest to you!

You will need to tell me who your partner is by February 21, give me a (brief!) outline of what you plan to do for your final project by March 7, and let me see a more substantial first draft of your final paper by April 25.

I realize some of you may have senior seminars around this time–if you do, make sure you get a good head start on your project for this class!

Final Grade

Your numerical grades will be converted to letter grades and finally Grade Points via the following cutoffs (see the UMM Catalog for more on Grades and grading policy):

Numerical	93%	90%	87%	83%	80%	77%	73%	70%	67%	60%
Letter	A	A-	B+	B	B-	C+	C	C-	D+	D
Grade Point	4.00	3.67	3.33	3.00	2.67	2.33	2.00	1.67	1.33	1.00

Expectations

- Cooperation is vital to your future success, which ever path you take. I encourage cooperation amongst students where ever possible, but the act of copying or other forms of cheating will not be tolerated. Any act of plagiarism that is detected will result is a mark of zero on the assignment for both parties.

  Cheating is a tricky concept in this course, since I am *actively encouraging* collaborations at every stage. If you feel uncomfortable with the progress of your group work, or do not really understand if something has been plagiarized or not, talk to me.

- There will be no late assignments accepted unless arrangements are made with me **prior** to the due date of the assignment.

- Solutions to problems should be presented at a level that is appropriate to an upper level university course. This means solutions should be written legibly, contain diagrams where appropriate, and should state the problem and **explain** the solution. **The presentation of solutions and analysis will be a significant factor in your final grade!**

  This is one of the most important aspects of the course–how well you are able to communicate with others. If you wish, you can use *Mathematica* to do more than numerical work for you–you can insert comments directly, and end up with a useful model for you to use for future projects. For an example of what I mean, take a look at **http://cda.mrs.umn.edu/~mcquarrb/Modeling/SocialMobility.nb**.

Academic and Other Assistance

If you have any special needs or requirements to help you succeed in the class, come and talk to me as soon as possible, or visit the appropriate University service yourself. Some UMM resources include:
The Academic Assistance Center www.mrs.umn.edu/services/dsoaac/aac/
Student Counseling www.mrs.umn.edu/services/counseling/
Disability Services www.mrs.umn.edu/services/dsoaac/dso

It is University policy to provide reasonable accommodations to students with disabilities. This publication/material is available in alternative formats to persons with disabilities upon request. Contact the instructor or the Disability Services office, 589–6178, Room 362 Briggs Library to discuss accommodation needs.

A Penultimate Thought

I am looking forward to an exciting and productive semester with you all! I am positive that we can all make the most of this opportunity to grow and learn. It should be a great deal of fun (and brain sweat).

Course Outline and <u>Tentative</u> Dates:

Week	Dates	Readings (completed before Monday)	Notes
		Optimization Models	
1	Jan 15 & 17	Introduction, 1.1 The Five–Step Method, 1.2 Sensitivity Analysis, 1.3 Sensitivity and Robustness.	
2	Jan 22 & 24	2.1 Unconstrained optimization, 2.2 Lagrange Multipliers.	
3	Jan 29 & 31	2.3 Sensitivity analysis for Lagrange Multipliers, 3.1 Newton's Method, Global vs Local.	Assignment 1 due Jan 29 in class
4	Feb 5 & 7	3.2 Random Search, 3.3 Linear Programming.	
		Dynamic Models	
5	Feb 12 & 14	4.1 Steady State Analysis, 4.2 Dynamical Systems, 4.3 Discrete Time Dynamical Systems.	Assignment 2 due Feb 12 in class
6	Feb 19 & 21	5.1 Eigenvalue Methods (Continuous), 5.2 Eigenvalue Methods (Discrete), 5.3 Phase Portraits.	Groups for final project chosen Feb 21
7	Feb 26 & 28	6.1 Introduction to Simulation, 6.2 Continuous Time Models.	Assignment 3 due Feb 26 in class
8	Mar 5 & 7	Runge Kutta Order Four (handout), 6.4 Chaos and Fractals.	Outline of final project due Mar 7
		Probability Models	
9	Mar 19 & 21	7.1 Discrete Probability Models, 7.2 Continuous Probability Models.	Assignment 4 due Mar 19 in class
10	Mar 26 & 28	8.1 Markov Chains, 8.2 Markov Processes, 8.3 Linear Regression.	Ideas for Variable Topics due Mar 28
11	Apr 2 & 4	9.1 Monte Carlo Simulation, 9.2 The Markov Property.	Assignment 5 due Apr 2 in class
12	Apr 9 & 11	9.3 Analytic Simulation.	
		Variable Topics Decided by Class	
13	Apr 16 & 18		Assignment 6 due Apr 16 in class
14	Apr 23 & 25		Draft of final project due Apr 25
15	Apr 30 May 2	Final Project Presentations Final Project Presentations	