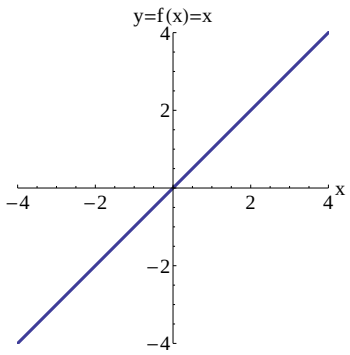
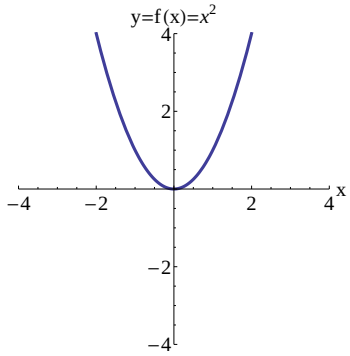
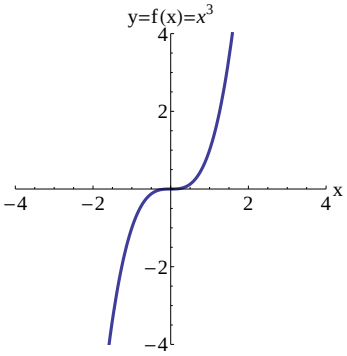


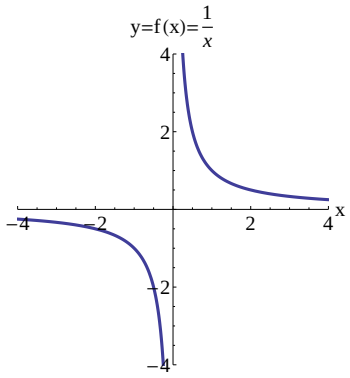
Identity Function



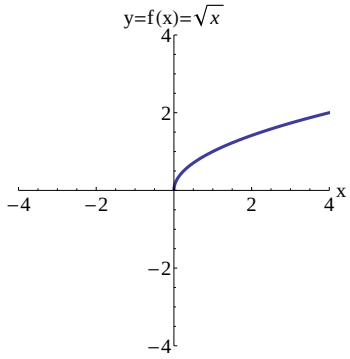
Squaring Function



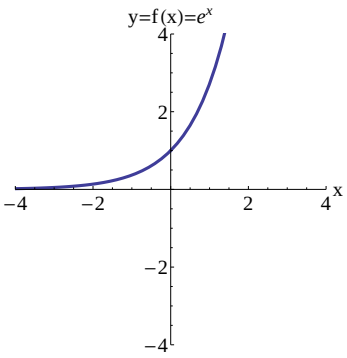
Cubing Function



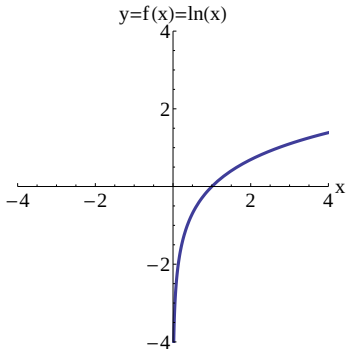
Inverse Function



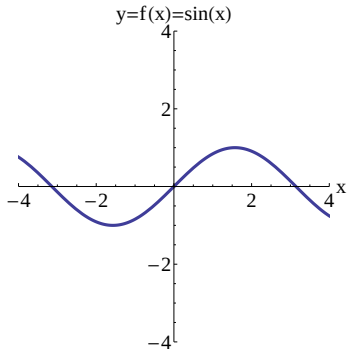
Square Root Function



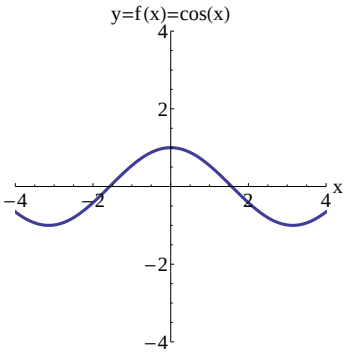
Exponential Function



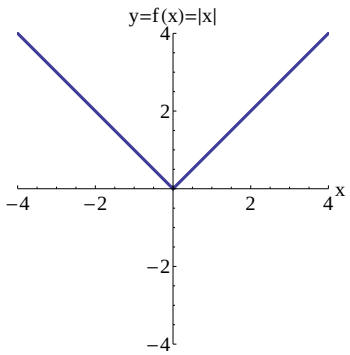
Natural Logarithmic Function



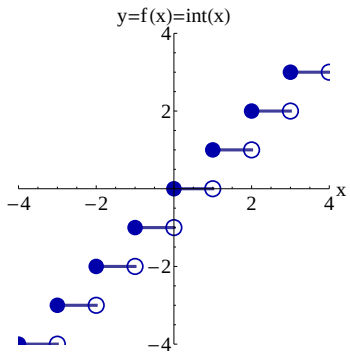
Sine Function



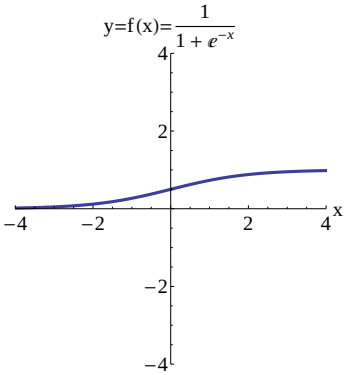
Cosine Function



Absolute Value Function



Greatest Integer Function



Logistic Function

Concepts: Familiarity with graphs of the 12 Basic Functions, and determining the properties of these functions from their graphs.

Questions: For the 12 Basic functions, use the sketches to answer the following (we will focus on the algebraic properties in the coming weeks).

1. Which have domain $x \in \mathbb{R}$ (another way of saying this is domain is $x \in (-\infty, \infty)$, or $-\infty < x < \infty$)?

   $f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, $f(x) = e^x$, $f(x) = \sin(x)$, $f(x) = \cos(x)$, $f(x) = |x|$, $f(x) = \text{int}(x)$, $f(x) = \dfrac{1}{1+e^{-x}}$

2. Which have domain $x \in [0, \infty)$ (domain $0 \leq x < \infty$)?

   $f(x) = \sqrt{x}$

3. Which have domain $x \in (0, \infty)$ (domain $0 < x < \infty$)?

   $f(x) = \ln(x)$

4. Which have range $y \in \mathbb{R}$ (range $-\infty < y < \infty$)?

   $f(x) = x$, $f(x) = x^3$, $f(x) = \ln(x)$

5. Which have range $y \in [0, \infty)$ (range $0 \leq y < \infty$)?

   $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = |x|$

6. Which have range $y \in (0, \infty)$ (range $0 < y < \infty$)?

   None.

7. Which have range $y \in (-\infty, 0) \cup (0, \infty)$ (range $-\infty < y < 0$ union with $0 < y < \infty$, which means $y \neq 0$)?

   $f(x) = \dfrac{1}{x}$

8. Which have range $y \in (0, 1)$ (range $0 < y < 1$)?

   $f(x) = \dfrac{1}{1+e^{-x}}$

9. Which have vertical asymptotes?

   $f(x) = \dfrac{1}{x}$ has a vertical asymptote at $x = 0$

   Aside: More formally, we may write:

   $\lim\limits_{x \to 0^+} \left(\dfrac{1}{x}\right) = \infty$ (read "the limit as x approaches 0 from the right of $1/x$ is infinity")

   $\lim\limits_{x \to 0^-} \left(\dfrac{1}{x}\right) = -\infty$ ("the limit as x approaches 0 from the left of $1/x$ is minus infinity").

   We will talk about these right and left handed limits more in the coming weeks. I am including them here to help us get comfortable with the notation.

   $f(x) = \ln(x)$ has a vertical asymptote at $x = 0$

   $\lim\limits_{x \to 0^+} \ln(x) = -\infty$ (read "the limit as x approaches 0 from the right of $\ln(x)$ is minus infinity")

10. Which have horizontal asymptotes?

$f(x) = \dfrac{1}{x}$ has a horizontal asymptote at $y = 0$

$\lim\limits_{x\to\infty}\left(\dfrac{1}{x}\right) = 0$ (read "the limit as x approaches infinity of $1/x$ is zero")
$\lim\limits_{x\to-\infty}\left(\dfrac{1}{x}\right) = 0$ (read "the limit as x approaches minus infinity of $1/x$ is zero")

$f(x) = e^x$ has a horizontal asymptote at $y = 0$

$\lim\limits_{x\to-\infty}(e^x) = 0$ (read "the limit as x approaches minus infinity of e^x is zero")

$f(x) = \dfrac{1}{1+e^{-x}}$ has a horizontal asymptote at $y = 0$ and $y = 1$

$\lim\limits_{x\to\infty}\left(\dfrac{1}{1+e^{-x}}\right) = 1$ (read "the limit as x approaches infinity of $\frac{1}{1+e^{-x}}$ is one")
$\lim\limits_{x\to-\infty}\left(\dfrac{1}{1+e^{-x}}\right) = 0$ (read "the limit as x approaches minus infinity of $\frac{1}{1+e^{-x}}$ is zero")

11. Which have local extrema?

$f(x) = x^2$ has a global minimum of $y = 0$ at $x = 0$
$f(x) = \sqrt{x}$ has a global minimum of $y = 0$ at $x = 0$
$f(x) = \sin(x)$ has an infinite number of global minimums and maximums
$f(x) = \cos(x)$ has an infinite number of global minimums and maximums
$f(x) = |x|$ has a global minimum of $y = 0$ at $x = 0$

12. Which are bounded below?

$f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = e^x$, $f(x) = \sin(x)$, $f(x) = \cos(x)$, $f(x) = |x|$, $f(x) = \dfrac{1}{1+e^{-x}}$

13. Which have discontinuities?

$f(x) = \dfrac{1}{x}$, $f(x) = \text{int}(x)$

14. Which are even ($f(-x) = f(x)$ for all x in domain)?

$f(x) = x^2$, $f(x) = \cos(x)$, $f(x) = |x|$

15. Which are odd ($f(-x) = -f(x)$ for all x in domain)?

$f(x) = x$, $f(x) = x^3$, $f(x) = \dfrac{1}{x}$, $f(x) = \sin(x)$

16. Which are increasing over the interval $x \in (-\infty, 0)$ (increasing for $-\infty < x < 0$)?

$f(x) = x$, $f(x) = x^3$, $f(x) = e^x$, $f(x) = \dfrac{1}{1+e^{-x}}$