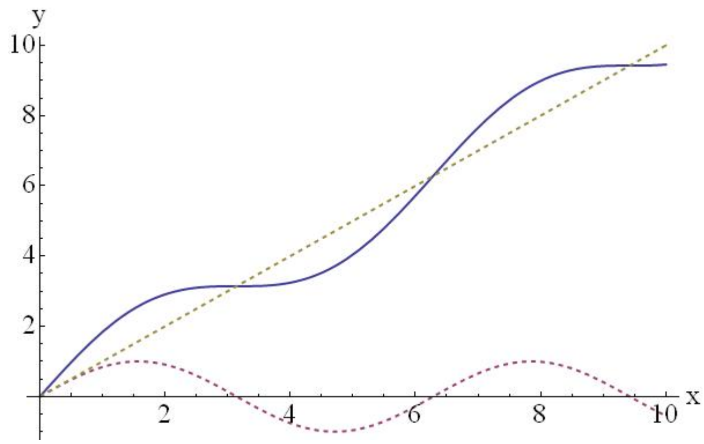
Concepts: Adding Trig Functions, Multiplying Trig Functions.

We obtain the graphs of the composite trig functions by combining two trig functions. In this section, simply try to get a feel for how sinusoids and other functions compose. Most of these sketches would be difficult to get by hand.

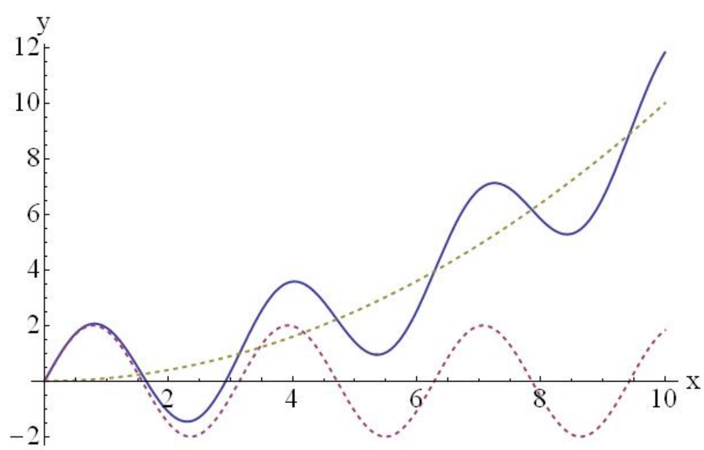
Graphical Addition: Sinusoid and a Linear Function

The solid line is the function $f(x) + g(x) = x + \sin x$. It is found by adding the y-coordinates of $f(x) = x$ and $g(x) = \sin x$. Notice that the result is an oscillation about the line $y = x$.



Graphical Addition: Sinusoid and a Quadratic Function

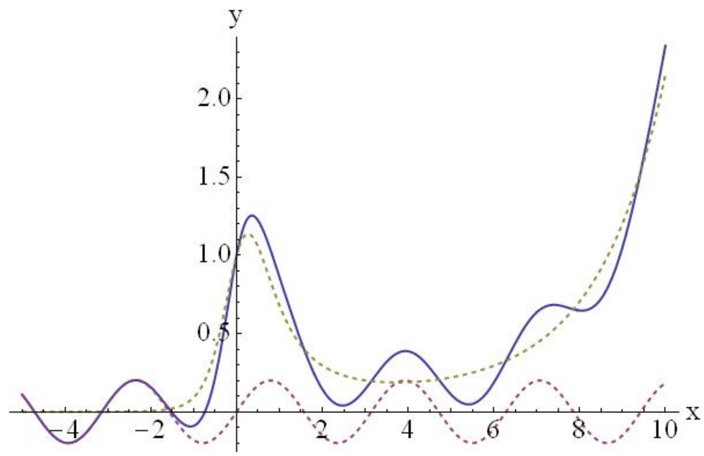
The solid line is the function $f(x) + g(x) = x^2/10 + 2 \sin 2x$.



When you add a sinusoid to a polynomial, you get a function which oscillates about the polynomial. This function is not periodic, although it does oscillate.

Graphical Addition: Sinusoid and a More Complicated Function

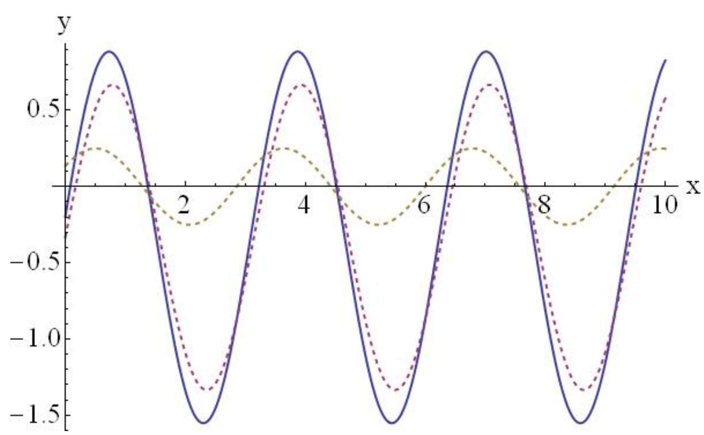
The solid line is the function $f(x) + g(x) = e^x/(x^2+1)^2 + \frac{1}{5}\sin 2x$.



When you add a sinusoid to a complicated function, you get a function which oscillates about the complicated function.

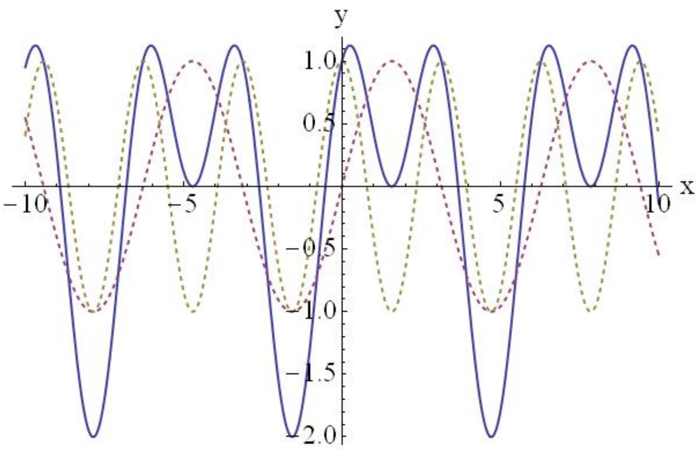
The Sum of Two Sinusoids is a Sinusoid if the Periods of the Sinusoids are The Same

The solid line is the function $f(x) + g(x) = \sin(2x) - \frac{1}{3} + \frac{1}{4}\cos(2x-1)$. Both sinusoids have period π, and the resulting function is also a sinusoid with period π.



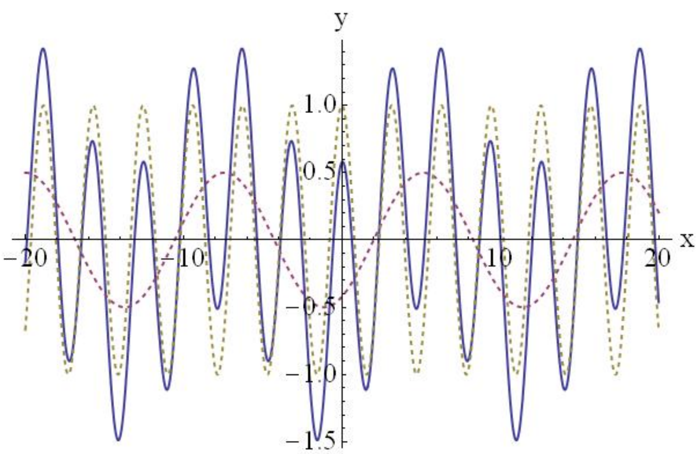
The Sum of Two Sinusoids is not a Sinusoid if the Periods of the Sinusoids are Different

The solid line is the function $f(x) + g(x) = \sin x + \cos(2x)$. One has period 2π, and the other has period π, and the resulting function is not a sinusoid. It is, however, periodic.



Periodic Functions: Verifying graphically, and algebraically

The function $\sin(x/2 - 1)$ has period 2π, and the $\cos(2x)$ has period π. We see that their sum (solid line) is not a sinusoid. It does appear, however, to be periodic, with a period of about $12 \sim 4\pi$.



To verify the period of $f(x) = \sin(x/2 - 1) + \cos 2x$ is 4π, we must do some algebra.

$$\begin{aligned} f(x + 4\pi) &= \sin((x + 4\pi)/2 - 1) + \cos 2(x + 4\pi) \\ &= \sin(x/2 - 1 + 2\pi) + \cos(2x + 8\pi) = \sin(x/2 - 1) + \cos(2x) = f(x) \end{aligned}$$

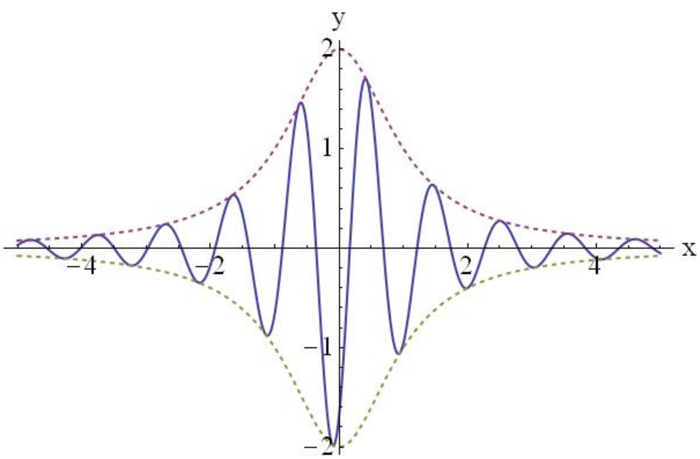
where the last step is true since $\sin\theta$ and $\cos\theta$ have period 2π.

Since we have shown $f(x + 4\pi) = f(x)$, the period is 4π or the period is an exact divisor of 4π (ie., smaller than 4π). The figure above suggests the period is not smaller than 4π, so the period must be 4π.

Multiplication: Damping and Beats

The graph of $y = f(x)g(x)$ where $g(x)$ is a sinusoid oscillates between the graphs of $f(x)$ and $-f(x)$. When the amplitude of the wave is reduced, this is referred to as *damping*.

For $f(x) = 2/(x^2+1)$ and $g(x) = \sin(6x-1)$, the function $y = f(x)g(x)$ (solid line) has envelopes given by $y = 2/(x^2+1)$ and $y = -2/(x^2+1)$ (dashed lines).



This even works even if $f(x)$ is another sinusoid. The one with the smaller period oscillates "inside" the sinusoid with the larger period. Here there is no damping, since the oscillations do not reduce in size, and this situation is called *beats*.

For $f(x) = \cos x$ (period 2π) and $g(x) = \sin(6x-1)$ (period $\pi/3$), the function $y = f(x)g(x)$ (solid line) has envelopes given by $y = \cos x$ and $y = -\cos x$ (dashed lines).