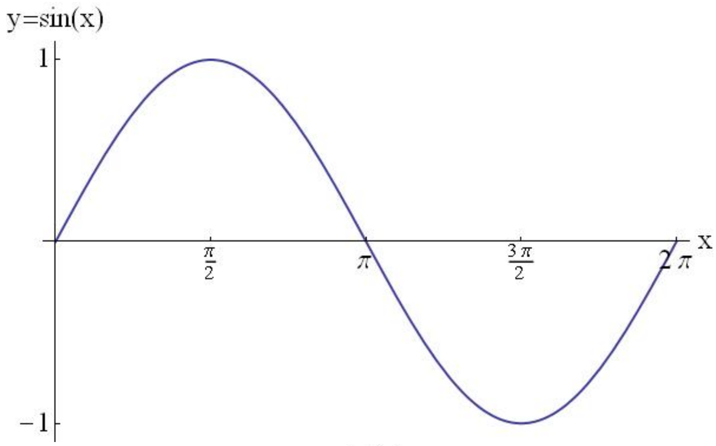
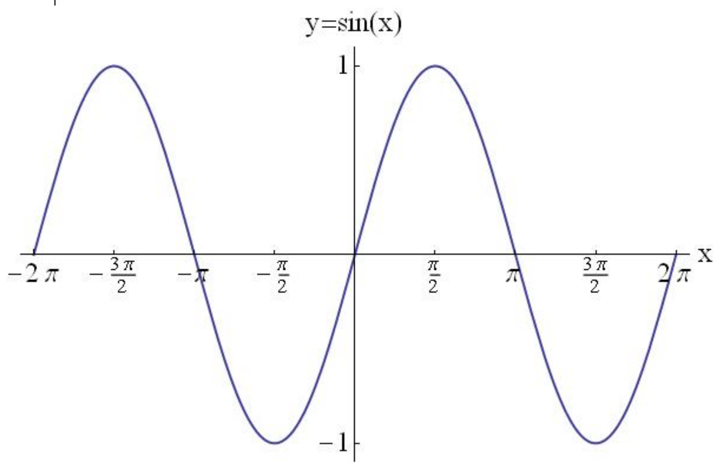
Concepts: Graphs of Sine, Cosine, Sinusoids, Terminology (amplitude, period, phase shift, frequency).

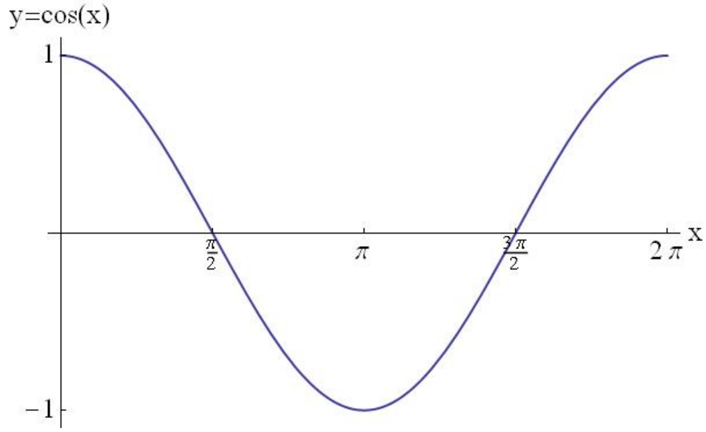
The Sine Function

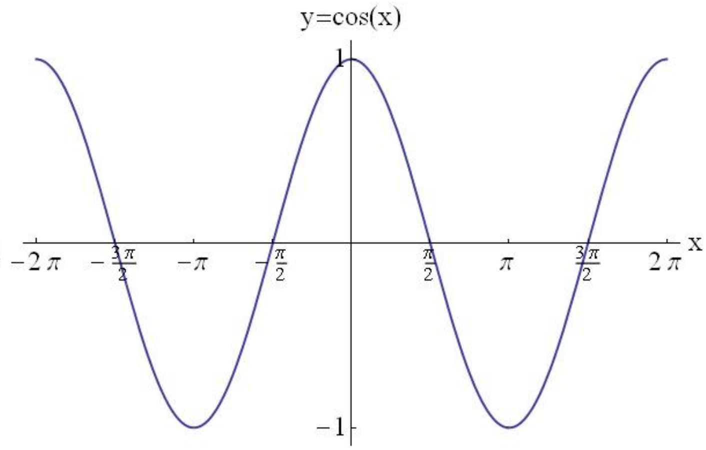




Domain: $x \in \mathbb{R}$
Range: $y \in [-1, 1]$
Continuity: continuous for all x
Increasing-decreasing behaviour: alternately increasing and decreasing
Symmetry: odd ($\sin(-x) = -\sin(x)$))
Boundedness: bounded above and below
Local Extrema: absolute max of $y = 1$, absolute min of $y = -1$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $+1$ and -1.
This is a periodic function with period 2π.

The Cosine Function





Domain: $x \in \mathbb{R}$
Range: $y \in [-1, 1]$
Continuity: continuous for all x
Increasing-decreasing behaviour: alternately increasing and decreasing
Symmetry: even ($\cos(-x) = \cos(x)$))
Boundedness: bounded above and below
Local Extrema: absolute max of $y = 1$, absolute min of $y = -1$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $+1$ and -1.
This is a periodic function with period 2π.

Terminology

The *amplitude* of $\sin\theta$ and $\cos\theta$ is 1. Graphically, this is half the height of the wave.

The *period* of $\sin\theta$ and $\cos\theta$ is length of one full cycle of the wave, which is 2π.

The *frequency* is the reciprocal of the period, so $\sin\theta$ and $\cos\theta$ have a frequency of $1/(2\pi)$. The frequency is the number of wave cycles the function completes in a unit interval.

Notice that $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$. This relation between the cosine and sine leads us to consider other functions that have the basic shape of a sine function, called *sinusoids*.

Sinusoids

Any function which can be written in the form

$$f(x) = a\sin(bx + c) + d$$

where a, b, c and d are constants (neither $a = 0$ nor $b = 0$) is called a —it sinusoid. Notice that a sinusoid is simply a (graphical or algebriac) transformation of the function $\sin x$. We will use this to understand the properties of sinusoids.

Terminology for Sinusoids

The *amplitude* of the sinusoid is $|a|$. Graphically, this is half the height of the wave.

The *period* of the sinusoid is length of one full cycle of the wave.

The *phase shift* is a measure of how the sinusoid is horizontally shifted from the original sine function.

Since the sine function $\sin\theta$ is periodic with period 2π, it completes one entire wave if $0 \le \theta \le 2\pi$.

Therefore, a sinusoid will complete one entire wave if (think of $\theta = bx + c$)

$$\begin{aligned} 0 \le \quad & bx + c & \le 2\pi \\ -c \le \quad & bx & \le 2\pi - c \\ -\frac{c}{b} \le \quad & x & \le \frac{2\pi}{b} - \frac{c}{b} \quad \text{if } b > 0 \end{aligned}$$

The sinusoid has period $2\pi/|b|$ and phase shift $-c/b$.

The *frequency* is the reciprocal of the period, so a sinusoid has a frequency of $|b|/(2\pi)$. The frequency is the number of wave cycles the function completes in a unit interval.

The sinusoid has a vertical shift of d.

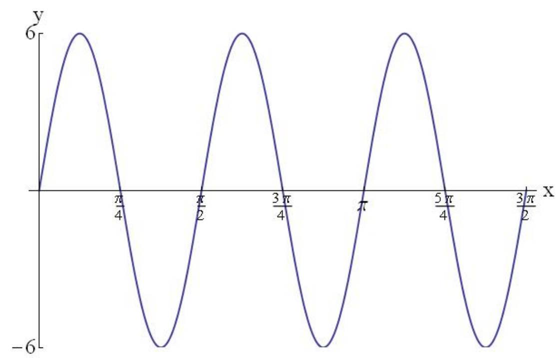
Analyse each sinusoid individually instead of trying to memorize all this, as the following examples show.

Example Graph three periods of the function $y = 6\sin(4x)$, using transformations and not calculators.

The function $\sin\theta$ has period 2π, which means it completes one wave for $0 \le \theta \le 2\pi$.
Our function will complete one entire wave in

$$0 \le \quad 4x \quad \le 2\pi$$
$$0 \le \quad x \quad \le \frac{2\pi}{4}$$
$$0 \le \quad x \quad \le \frac{\pi}{2}$$

The period of the function is $\pi/2$, so will will need to plot the function for $0 \le x \le 3\pi/2$ to get three periods.
The amplitude of the wave will be 6 units. From this we can get the sketch.

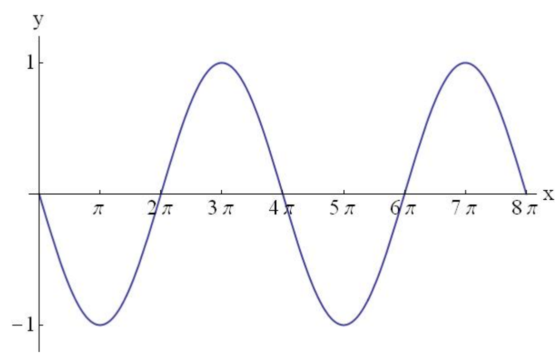


Example Graph two periods of the function $y = -\sin\left(\frac{x}{2}\right)$, using transformations and not calculators.

The function $\sin\theta$ has period 2π, which means it completes one wave for $0 \le \theta \le 2\pi$.
Our function will complete one entire wave in

$$0 \le \quad \tfrac{x}{2} \quad \le 2\pi$$
$$0 \le \quad x \quad \le 4\pi$$

The period of the function is 4π, so will will need to plot the function for $0 \le x \le 8\pi$ to get two periods.
The amplitude of the wave will be 1 units. The minus sign out front means the graph will be reflected about the x-axis.
From this we can get the sketch.



Example Find the amplitude, period, frequency, phase shift and vertical shift of the sinusoid

$$y = 3\cos(-2x+1) - 2.$$

Note: Although this is a cosine, it is still considered a sinusoid, and we can sketch it by thinking of translating the graph of $\cos x$ rather than $\sin x$.

The amplitude of the sinusoid is $|a| = 3$.

Since the cosine function is periodic with period 2π, it completes one entire wave if $0 \le \theta \le 2\pi$.

Therefore, the sinusoid will complete one entire wave if

$$\begin{aligned} 0 \le \quad & -2x+1 && \le 2\pi \\ -1 \le \quad & -2x && \le 2\pi - 1 \\ \frac{1}{2} \ge \quad & x && \ge -\pi + \frac{1}{2} \\ \frac{1}{2} - \pi \le \quad & x && \le \frac{1}{2} \end{aligned}$$

The sinusoid has period π and phase shift $1/2$.

The frequency is the reciprocal of the period, so a sinusoid has a frequency of $1/\pi$.

The sinusoid has a vertical shift of -2.

We can sketch the sinusoid once we know all of this information. Here is a sketch of one period of the cosine function, and one period of the sinusoid.