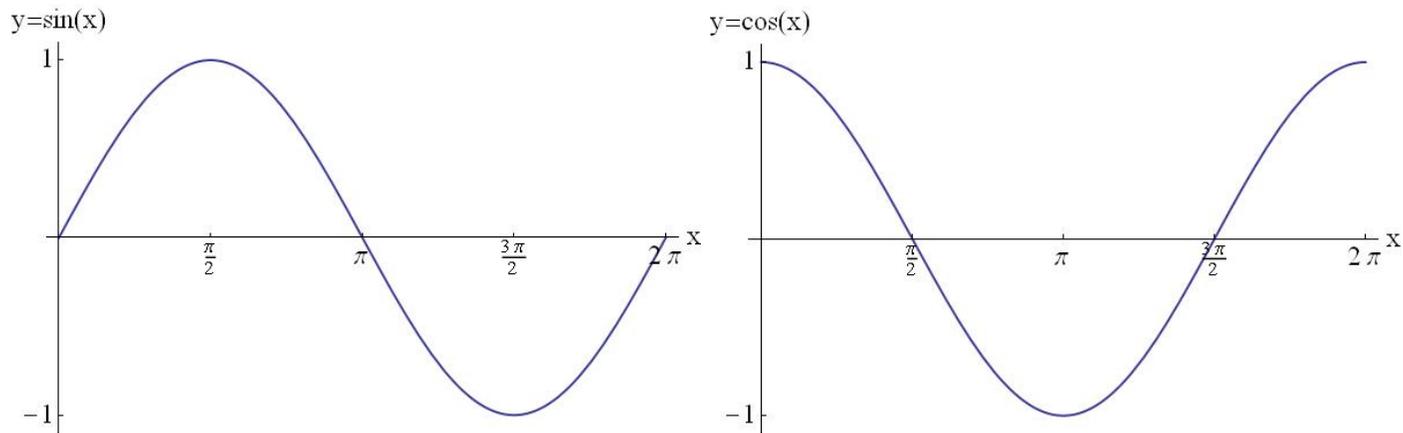**Concepts:** Graphs of Tangent, Cotangent, Secant, and Cosecant.

We obtain the graphs of the other trig functions by thinking about how they relate to the $\sin x$ and $\cos x$.

The Sine and Cosine Functions

The Tangent Function

The tangent function is $\tan x = \dfrac{\sin x}{\cos x}$. We use this to get the sketch.

It will have zeros where the sine function has zeros, and vertical asymptotes where the cosine function has zeros.

It will look like the sine function where the cosine is essentially equal to 1, which is when x is near 0 or 2π.

I have included the information about which Quadrant we are in, since that can help us get the correct positive/negative behaviour of the tangent function.

The graph on the left has the graph of $\sin x$ (dashed), whose zeros gives us the zeros of $\tan x$.

The graph on the right has the graph of $\cos x$ (dashed), whose zeros gives us the vertical asymptotes of $\tan x$.
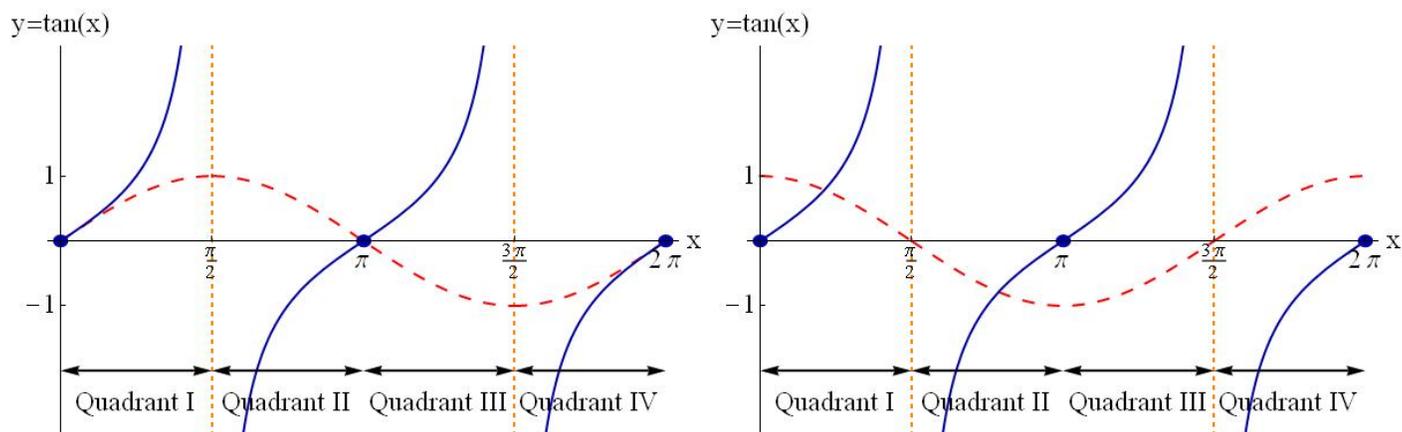
Domain: $x \in \mathbb{R}$ except $x = \dfrac{\pi}{2} + k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
Range: $y \in \mathbb{R}$
Continuity: continuous on its domain
Increasing-decreasing behaviour: increasing on each interval in its domain
Symmetry: odd $(\tan(-x) = -\tan(x)))$
Boundedness: not bounded
Local Extrema: none
Horizontal Asymptotes: none
Vertical Asymptotes: $x = \dfrac{\pi}{2} + k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period π.

The Cotangent Function

The tangent function is $\cot x = \dfrac{\cos x}{\sin x}$.

It will have zeros where the cosine function has zeros, and vertical asymptotes where the sine function has zeros.

It will look like the cosine function where the sine is essentially equal to 1, which is when x is near $\pi/2$.

The graph on the left has the graph of $\sin x$ (dashed), whose zeros gives us the vertical asymptotes of $\cot x$.

The graph on the right has the graph of $\cos x$ (dashed), whose zeros gives us the zeros of $\cot x$.
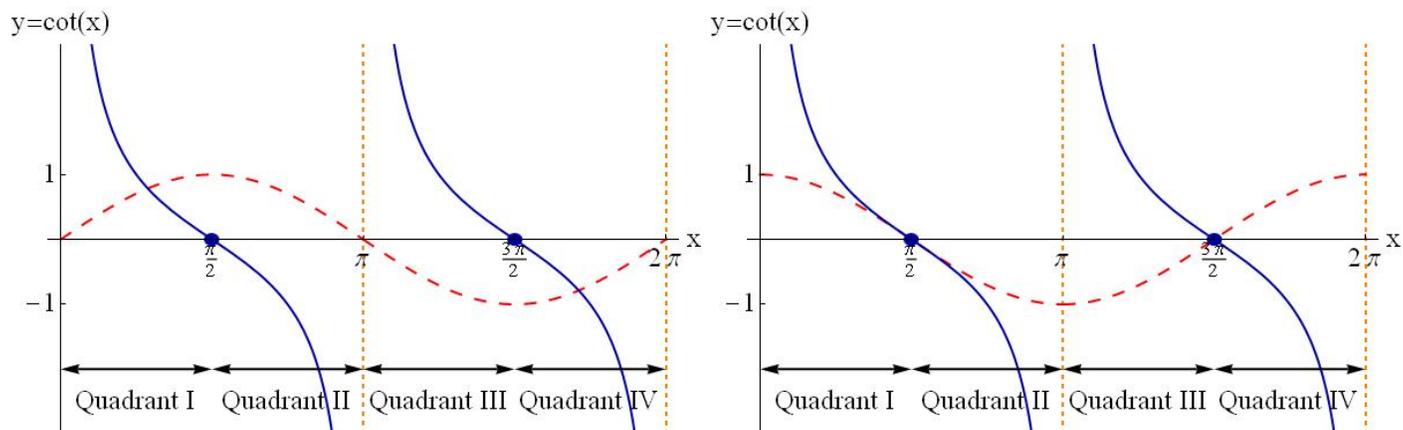
Domain: $x \in \mathbb{R}$ except $x = k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
Range: $y \in \mathbb{R}$
Continuity: continuous on its domain
Increasing-decreasing behaviour: decreasing on each interval in its domain
Symmetry: odd $(\cot(-x) = -\cot(x)))$
Boundedness: not bounded
Local Extrema: none
Horizontal Asymptotes: none
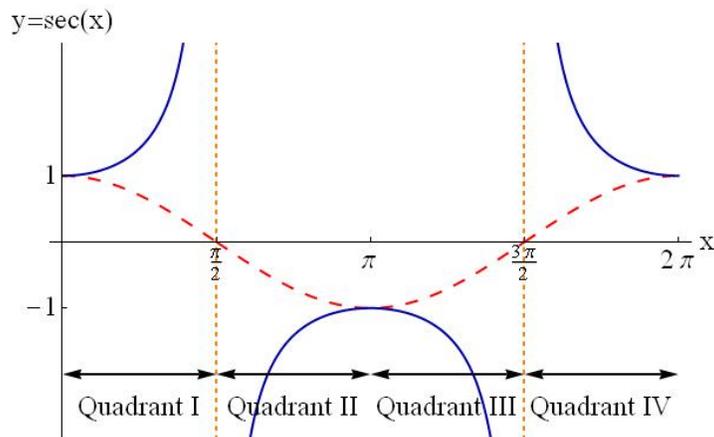Vertical Asymptotes: $x = k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period π.

The Secant Function

The secant function is $\sec x = \dfrac{1}{\cos x}$.

It will have vertical asymptotes where the cosine function has zeros. It will have no zeros.

Domain: $x \in \mathbb{R}$ except $x = \dfrac{\pi}{2} + k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$

Range: $y \in (-\infty, -1] \cup [1, \infty)$

Continuity: continuous on its domain

Increasing-decreasing behaviour: increases and decreases on each interval in its domain

Symmetry: even $(\sec(-x) = \sec(x)))$

Boundedness: not bounded

Local Extrema: local min at $x = 2k\pi$, local max at $x = (2k+1)\pi$, $k = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Horizontal Asymptotes: none

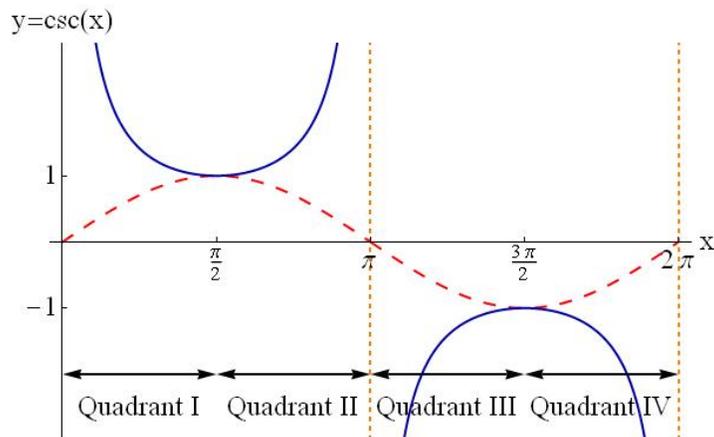Vertical Asymptotes: $x = \dfrac{\pi}{2} + k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$

End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$. This is a periodic function with period 2π.

The Cosecant Function

The cosecant function is $\csc x = \dfrac{1}{\sin x}$.

It will have vertical asymptotes where the sine function has zeros. It will have no zeros.

Domain: $x \in \mathbb{R}$ except $x = k\pi$, $k = \dots, -3, 2, 1, 0, 1, 2, 3, \dots$
Range: $y \in (-\infty, -1] \cup [1, \infty)$
Continuity: continuous on its domain
Increasing-decreasing behaviour: increases and decreases on each interval in its domain
Symmetry: odd $(\csc(-x) = -\csc(x)))$
Boundedness: not bounded
Local Extrema: local min at $x = \pi/2 + 2k\pi$, local max at $x = 3\pi/2 + 2k\pi$, $k = \dots, -3, -2, -1, 0, 1, 2, 3, \dots$
Horizontal Asymptotes: none
Vertical Asymptotes: $x = k\pi$, $k = \dots, -3, 2, 1, 0, 1, 2, 3, \dots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period 2π.

Example Solve the equation $\csc t = 2$ in the interval $\pi/2 \le t \le \pi$. You should not need a calculator to solve this problem.

Solve this using reference triangles, so you can see what that would look like.

The cosecant is positive where the sine is positive, which means we are in either Quadrant I or II. With no other information, this is all we can say. Let's assume we are in Quadrant I and proceed with our solution. Since we are told not to use a calculator, we expect that the reference triangle will be one of the two special triangles, a 45-45-90 or a 30-60-90, or the angle will be a quadrantal angle.

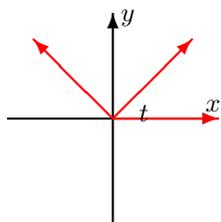$$\csc t = 2 = \frac{2}{1} = \frac{\text{hyp}}{\text{opp}}$$
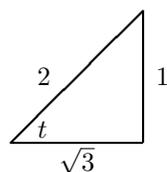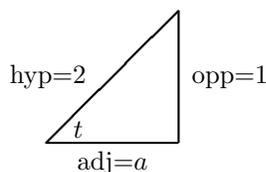
Using the Pythagorean Theorem, we see $2^2 = a^2 + 1^2 \longrightarrow a = \sqrt{3}$.
This triangle results from the equilateral triangle:
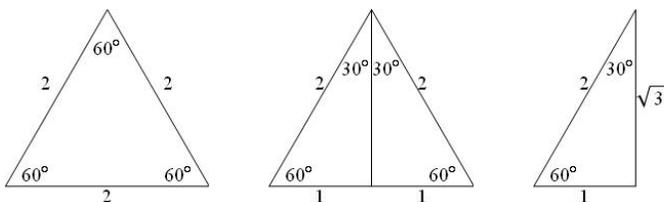
We see the angle is $30° = \pi/6$ radians. The cosecant is periodic with period 2π, so another solution is $t = \pi/6 + 2\pi = 13\pi/6$. This is also not in the interval asked for.
We still have a solution in Quadrant II, which we haven't dealt with yet. Let's do that now.

There is a solution in Quadrant II: $\pi - \pi/6 = 5\pi/6$. Since this is in the interval asked for, we have found the solution. Here is a sketch of the cosecant function so you can see the solution makes sense