Concepts: power function, monomial function, direct variation, inverse variation, properties of cubing, square root, and reciprocal functions, transforming the cubing, square root, and reciprocal functions.

Power Functions

Any function that can be written in the form

$$f(x) = kx^a$$

where k and a are nonzero constants is a *power function.*

Nomenclature:
a is the power,
k is the proportionality constant, or variation constant.

If $a > 0$ the power function describes a direct variation.
If $a < 0$ the power function describes an inverse variation.

Any function that can be written in the form

$$f(x) = kx^n$$

where k is a nonzero constant and n is a whole number $(0, 1, 2, 3, 4, \dots)$ is a *monomial function.*

Every polynomial function is a sum of monomial functions.

Direct Variations

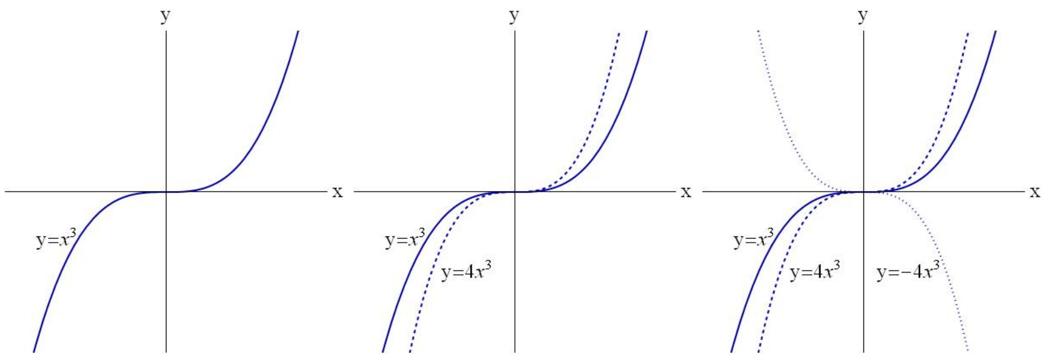
Basic Function: Cubing Function $f(x) = x^3$

The cubing function is a monomial function with direct variation.

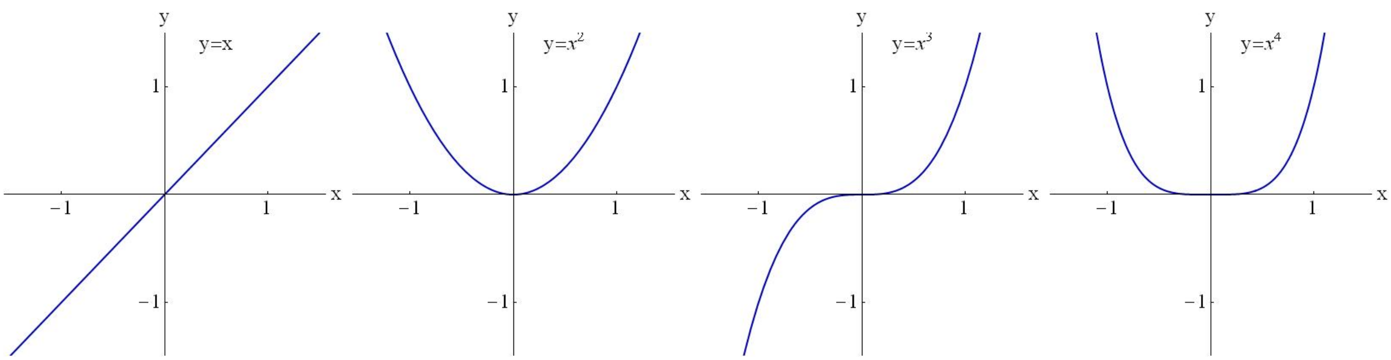
Domain: $x \in \mathbb{R}$
Range: $x \in \mathbb{R}$
Continuity: continuous for all x
Increasing-decreasing behaviour: increasing for all x
Symmetry: odd
Boundedness: not bounded
Local Extrema: none
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: $\lim_{x \to -\infty} x^3 = -\infty$ and $\lim_{x \to \infty} x^3 = \infty$

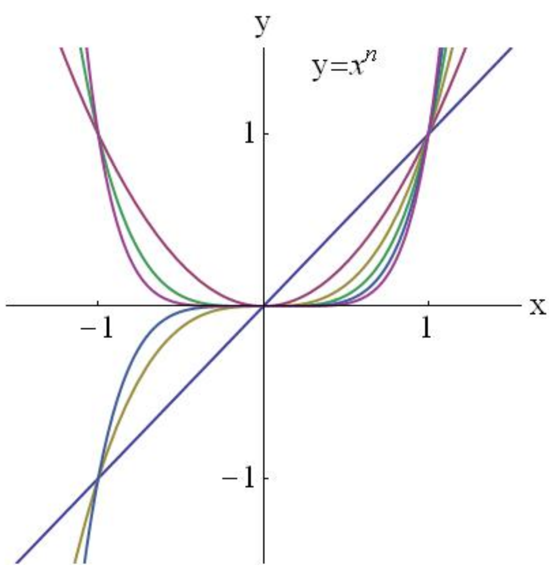
Transforming Cubing Function

Basic function: $y = x^3$ (solid line)
Vertical stretch of 4 units: $y = 4x^3$ (dashed line)
Reflect about x-axis: $y = -4x^3$ (dotted line)



Graphing Monomial Functions





In all cases, for a monomial function $f(x) = kx^n$, <u>with constant of variation $k > 0$</u>, we have $\lim_{x\to\infty} kx^n = \infty$. In sketches on the left, $k = 1$.

If the power n is odd, then we have $\lim_{x\to-\infty} kx^n = -\infty$.

If the power n is even, then we have $\lim_{x\to-\infty} kx^n = \infty$.

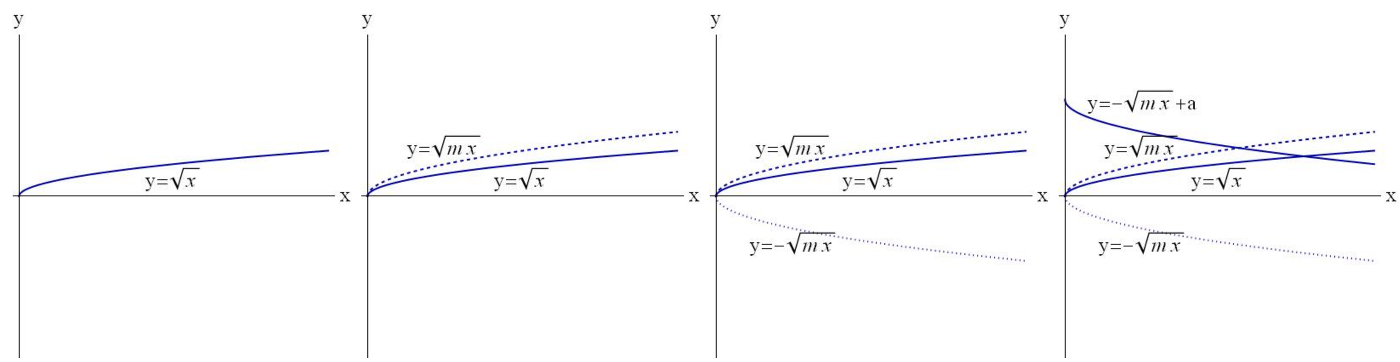
Basic Function: Square Root Function $f(x) = x^{1/2} = \sqrt{x}$

The square root function is not a monomial, however, it is a power function with direct variation.

Domain: $x \in [0, \infty)$
Range: $x \in [0, \infty)$
Continuity: continuous for all x in its domain
Increasing-decreasing behaviour: increasing for all x
Symmetry: none
Boundedness: bounded below but not bounded above
Local Extrema: local minimum at $x = 0$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: $\lim_{x\to-\infty} \sqrt{x} = \infty$

Transforming Square Root Function

Basic function: $y = \sqrt{x}$ (solid line)
Horizontal compression of m units ($m > 1$): $y = \sqrt{mx}$ (dashed line)
Reflect about x-axis: $y = -\sqrt{mx}$ (dotted line)
Move up a units ($a > 0$): $y = -\sqrt{mx} + a$ (solid line)



Inverse Variations

Basic Function: Reciprocal Function $f(x) = x^{-1}$

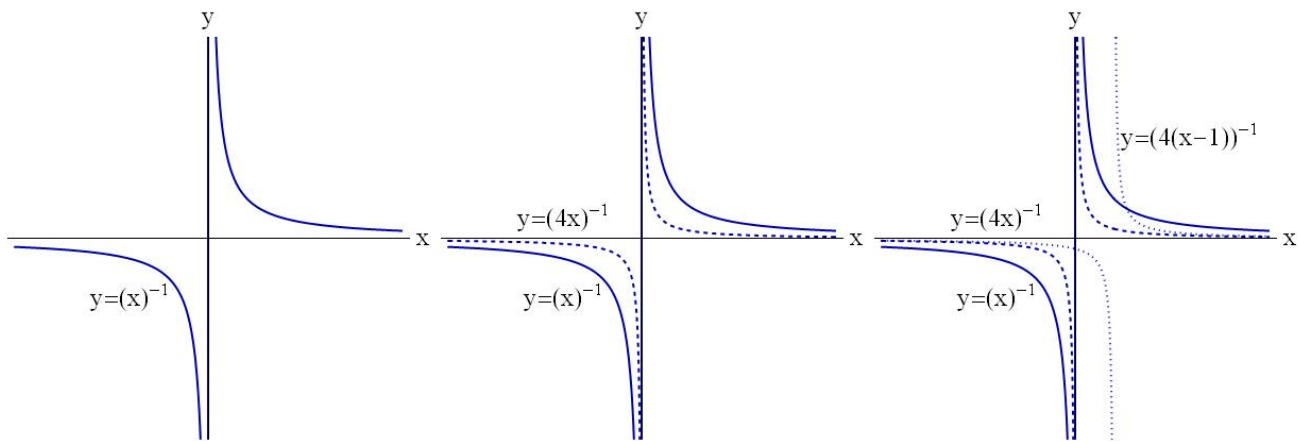
The reciprocal function is a power function with inverse variation.

Domain: $x \in (-\infty, 0) \cup (0, \infty)$
Range: $y \in (-\infty, 0) \cup (0, \infty)$
Continuity: discontinuous at $x = 0$
Increasing-decreasing behaviour: decreasing on $(-\infty, 0)$, increasing on $(0, \infty)$
Symmetry: odd
Boundedness: not bounded
Local Extrema: none
Horizontal Asymptotes: $y = 0$
Vertical Asymptotes: none
End behaviour: $\lim_{x\to-\infty} x^{-1} = 0$ and $\lim_{x\to\infty} x^{-1} = 0$

We will take a closer look at the reciprocal function in Section 2.7.

Transforming Reciprocal Function

Basic function: $y = x^{-1}$ (solid line)
Horizontal compression of 4 units: $y = (4x)^{-1}$ (dashed line)
Shift right 1 unit: $y = (4(x-1))^{-1} = \frac{1}{4x-4}$ (dotted line)



Note: the power function $f(x) = kx^a$ is not defined for $x < 0$ if a is irrational (like $a = \pi$) or if you need to take a square root.