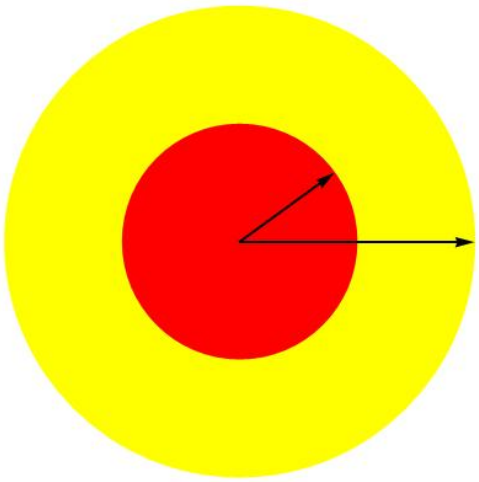
Questions

1. The concentric circles on an archery target are 6 inches apart. The inner circle (red) has a perimeter of 37.7 inches. What is the perimeter of the next largest (yellow) circle?

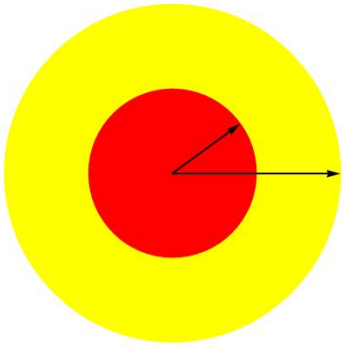


2. A radial arm saw has a circular cutting blade with a diameter of 10 inches. It spins at 2000 rpm. If there are 12 cutting teeth per inch on the cutting blade, how many teeth cross the cutting surface each second?

3. It takes ten identical pieces to form a circular track for a pair of toy racing cars. If the inside arc of each piece is 3.4 inches shorter than the outside arc, what is the width of the track?

Solutions

1. The concentric circles on an archery target are 6 inches apart. The inner circle (red) has a perimeter of 37.7 inches. What is the perimeter of the next largest (yellow) circle?



The key to solving this problem is to introduce your own notation, and start trying to find connections.

The outer (yellow) circle has radius r_2 and perimeter $p_2 = 2\pi r_2$ inches.
The perimeter p_2 is what we want to find.
The inner (red) circle has radius r_1 and perimeter $p_1 = 37.7 = 2\pi r_1$ inches.
Since the circles are six inches apart, $r_2 - r_1 = 6$.

Subtracting the first two equations above, we find:

$$\begin{array}{lcl} p_2 & = & 2\pi r_2 \\ 37.7 & = & 2\pi r_1 \\ \hline p_2 - 37.7 & = & 2\pi(r_2 - r_1) \end{array}$$

Solving this equation for the unknown p_2, we find:

$$p_2 = 2\pi(r_2 - r_1) + 37.7 = 2\pi(6) + 37.7 \sim 75.3991 \text{ inches.}$$

2. A radial arm saw has a circular cutting blade with a diameter of 10 inches. It spins at 2000 rpm. If there are 12 cutting teeth per inch on the cutting blade, how many teeth cross the cutting surface each second?

First, we need to find out how many teeth there are on the entire blade. The radius of the blade is half the diameter, so $r = 10/2 = 5$ inches.

The circumference of the blade is given by $p = 2\pi r = 2\pi r = 10\pi$.

Since there are 12 teeth per inch, the number of teeth on the blade is

$$\text{(number of teeth on the blade)} = \text{(number of teeth per inch)(circumference of blade)} = (12)(10\pi) = 120\pi \sim 376.991$$

There are 377 cutting teeth total on the blade. One is slightly smaller that the others due to irrational number π!

If there are 2000 rpm (revolutions per minute) that means there are

$$2000\frac{\text{revolutions}}{\text{minute}} \cdot \left(\frac{1 \text{ minute}}{60 \text{ seconds}}\right) = \frac{100}{3}\frac{\text{revolutions}}{\text{second}}.$$

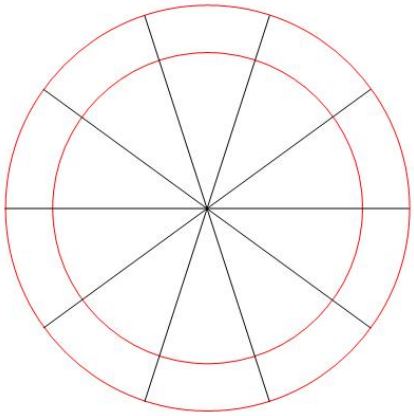
Note that the quantity in brackets is equal to 1. Multiplying by 1 is how we can change the units in a problem.

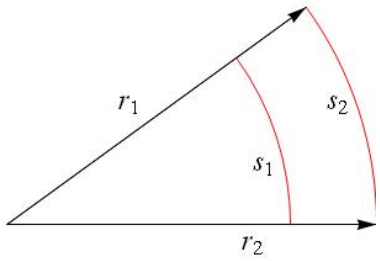
The number of teeth crossing the cutting surface each second is given by

$$\frac{100}{3}\frac{\text{revolutions}}{\text{second}} \cdot 120\pi \text{ teeth per revolution } = 4000\pi\frac{\text{teeth}}{\text{second}} \sim 12566.4\frac{\text{teeth}}{\text{second}}.$$

3. It takes ten identical pieces to form a circular track for a pair of toy racing cars. If the inside arc of each piece is 3.4 inches shorter than the outside arc, what is the width of the track?

We definitely need a diagram and some notation to help us solve this problem. Here is how the track lays out:





The "slice" on the left introduces some notation:

The outer circle has radius r_2 and circumference $2\pi r_2 = 10s_2$.
The inner circle has radius r_1 and circumference $2\pi r_1 = 10s_1$.
The width of the track is going to be $r_2 - r_1$, which is what we need to find.
The relationship we are given between the inner and outer arc lengths of a piece of track means $s_2 = s_1 + 3.4$, or $s_2 - s_1 = 3.4$.

Subtracting the first two equations above, we find:

$$\begin{array}{lcl} 2\pi r_2 & = & 10s_2 \\ 2\pi r_1 & = & 10s_1 \\ \hline 2\pi(r_2 - r_1) & = & 10(s_2 - s_1) \end{array}$$

Solving this equation for the unknown we seek, we find:

$$\text{width of track } = (r_2 - r_1) = \frac{5}{\pi}(s_2 - s_1) = \frac{5}{\pi}(3.4) = 5.411 \text{ inches}.$$