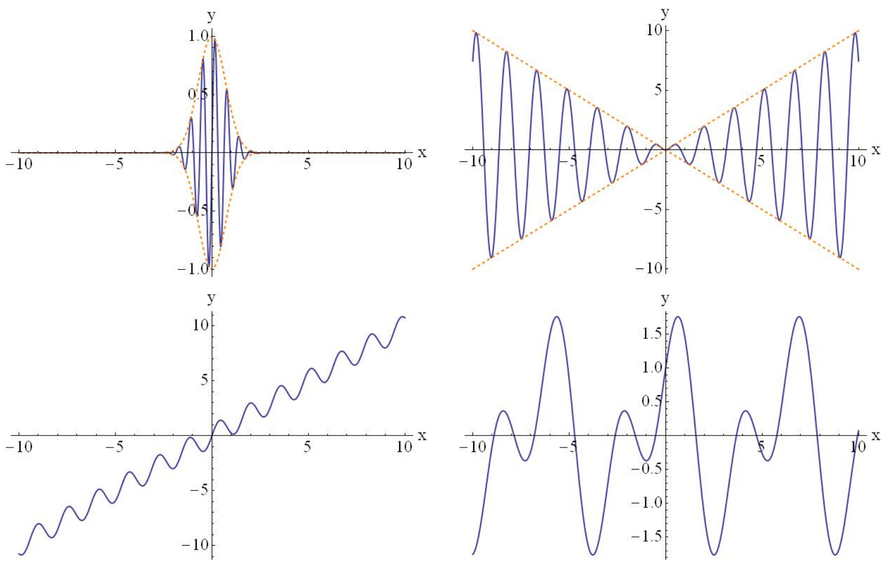
Questions

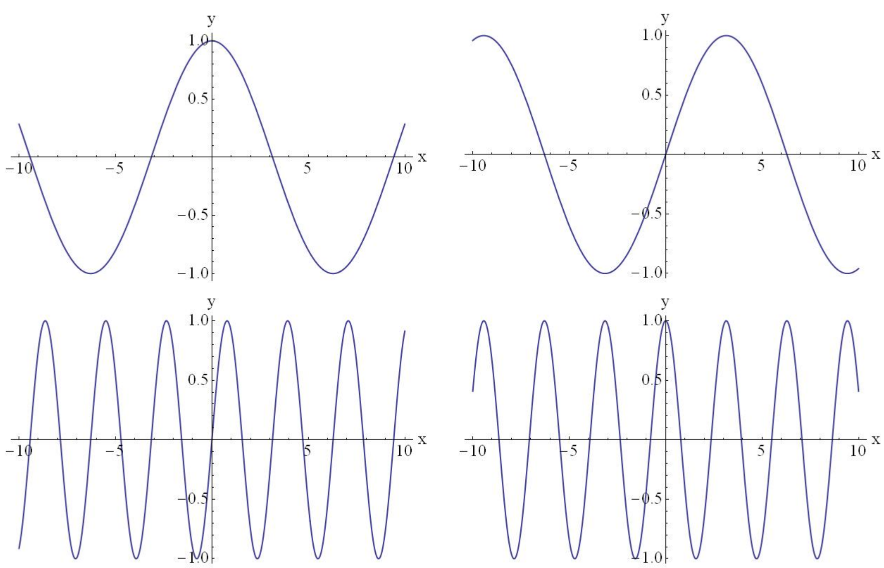
1. Match the function with its graph

(a) $y = x + \sin 4x$ (b) $y = x \sin 4x$ (c) $y = \cos x + \sin 2x$ (d) $y = e^{-x^2} \sin 10x$



2. Match the function with its graph

(a) $y = \sin 2x$ (b) $y = \sin(x/2)$ (c) $y = \cos 2x$ (d) $y = \cos(x/2)$



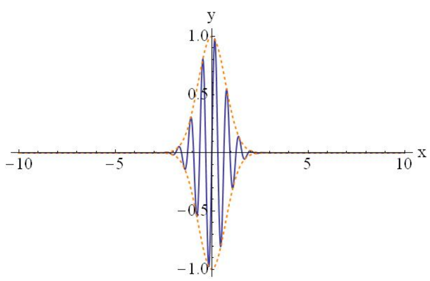
3. Does the function $f(x) = \left(\frac{1}{4}\right)^x \sin(\pi x + \pi/3)$, $x \in [0, \infty)$ exhibit damping? If so, identify the damping factor and tell if the damping occurs as $x \to \infty$ or $x \to 0$.

4. Algebraically show the function $y = \sin^3 x$ is a periodic function with period at least 2π.
Do you think $y = \sin(x^3)$ is periodic? Why or why not?

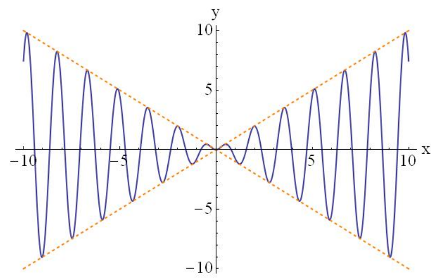
Solutions

1. Match the function with its graph

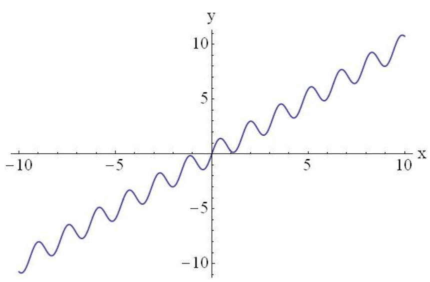
(a) $y = x + \sin 4x$ (b) $y = x \sin 4x$ (c) $y = \cos x + \sin 2x$ (d) $y = e^{-x^2} \sin 10x$



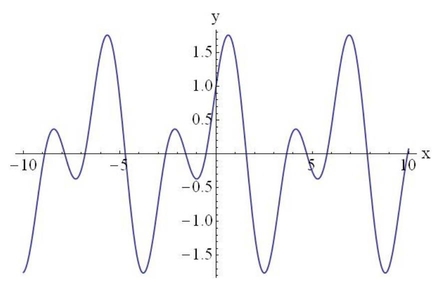
(d)



(b)



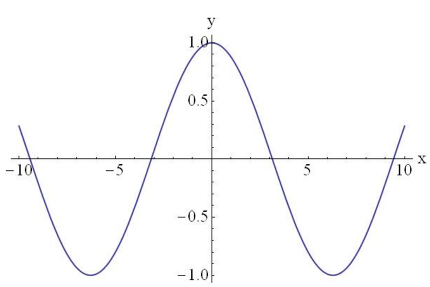
(a)



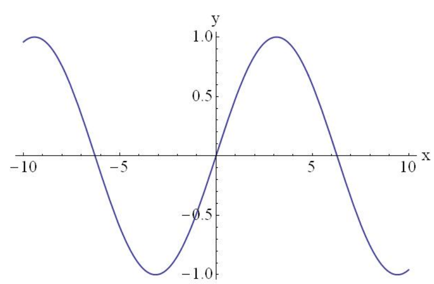
(c)

2. Match the function with its graph

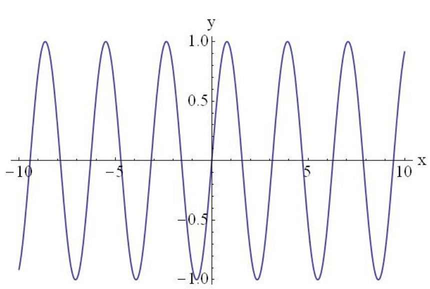
(a) $y = \sin 2x$ (b) $y = \sin(x/2)$ (c) $y = \cos 2x$ (d) $y = \cos(x/2)$



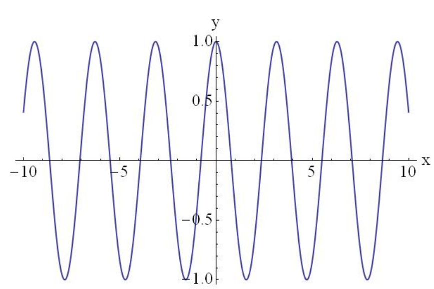
(d)



(b)



(a)



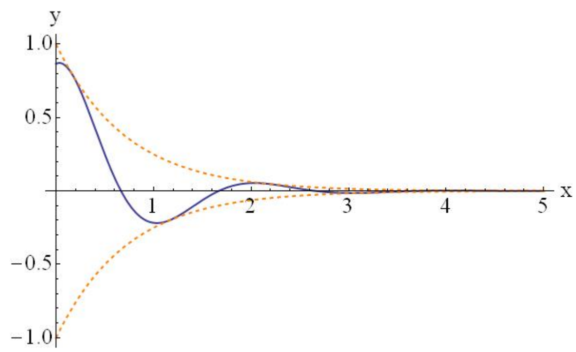
(c)

3. Does the function $f(x) = \left(\frac{1}{4}\right)^x \sin(\pi x + \pi/3)$, $x \in [0, \infty)$ exhibit damping? If so, identify the damping factor and tell if the damping occurs as $x \to \infty$ or $x \to 0$.

The factor $\left(\frac{1}{4}\right)^x$ will determine if there is damping. This is an exponential with base $b < 1$, so it will approach zero as $x \to \infty$, so it is a damping factor.

The function f will experience damping as $x \to \infty$, and oscillate between the envelope functions $y = \pm \left(\frac{1}{4}\right)^x$.

Here is a sketch (done by computer) that verifies our conclusions:



4. Algebraically show the function $y = \sin^3 x$ is a periodic function with period at least 2π.
Do you think $y = \sin(x^3)$ is periodic? Why or why not?
Let $f(x) = \sin^3 x$.

$$\begin{aligned} f(x + 2\pi) &= \sin^3(x + 2\pi) = (\sin(x + 2\pi))^3 \\ &= (\sin(x))^3 = \sin^3 x \\ &= f(x) \end{aligned}$$

So it is periodic, with periodic at least 2π. We would need to sketch to determine if the period was less than 2π. As the computer sketch below shows, the period of $y = \sin^3 x$ is 2π.

I would not expect $y = \sin(x^3)$ to be periodic, although I would expect it to oscillate between -1 and 1.

Why? Think of what happens to the angle x^3 as x increases. Since the quantity x^3 will move around the unit circle at an ever increasing rate (remember, x^3 is the angle!), I would expect the graph of $\sin(x^3)$ to move between -1 and 1 more rapidly as x increases. This is in fact exactly what happens for the graph of $y = \sin(x^3)$, as the following computer sketch shows!