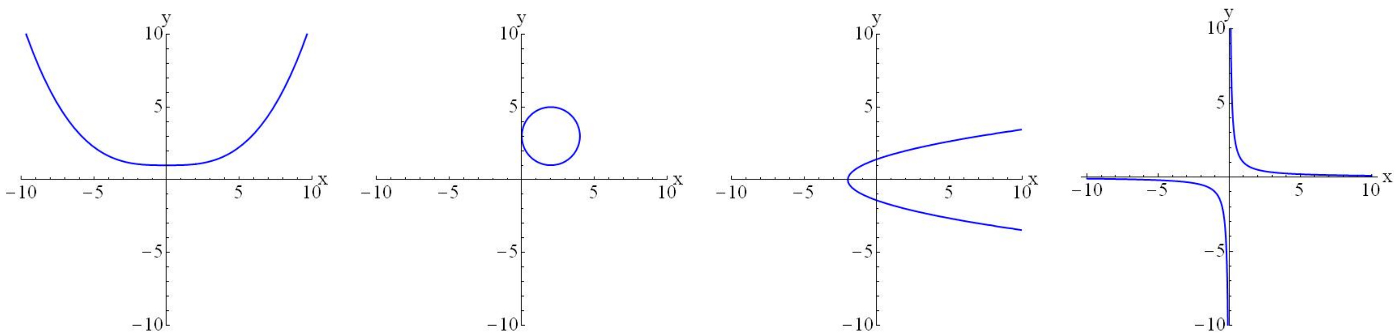
Questions

1. Which of the following set of points represents a function?



(a) (b) (c) (d)

2. Determine, using algebra, if the function $f(x) = \dfrac{12x}{1 - x^2}$ is odd, even, or neither odd nor even.

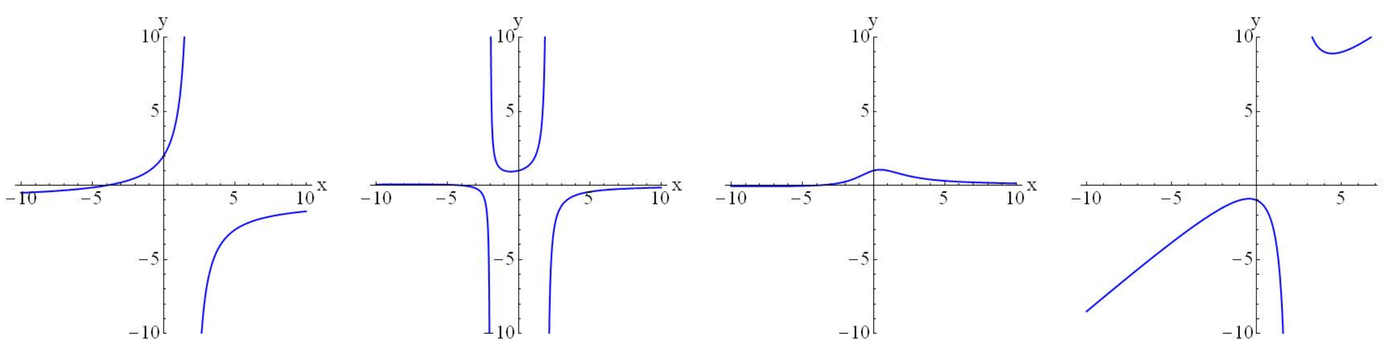
3. Determine, using algebra, if the function $f(x) = x^4 + x$ is odd, even, or neither odd nor even.

4. Find the domain of the function $f(x) = \dfrac{1}{x-2} + \sqrt{x^2 - 1}$.

5. Find the range of the function $f(x) = 5 + \sqrt{4 - x}$.

6. Match the function to its graph

(i) $y = \dfrac{x^2+2}{x-2}$ (ii) $y = \dfrac{x+4}{4+x^2}$ (iii) $y = \dfrac{x+4}{4-x^2}$ (iv) $y = \dfrac{x+4}{2-x}$

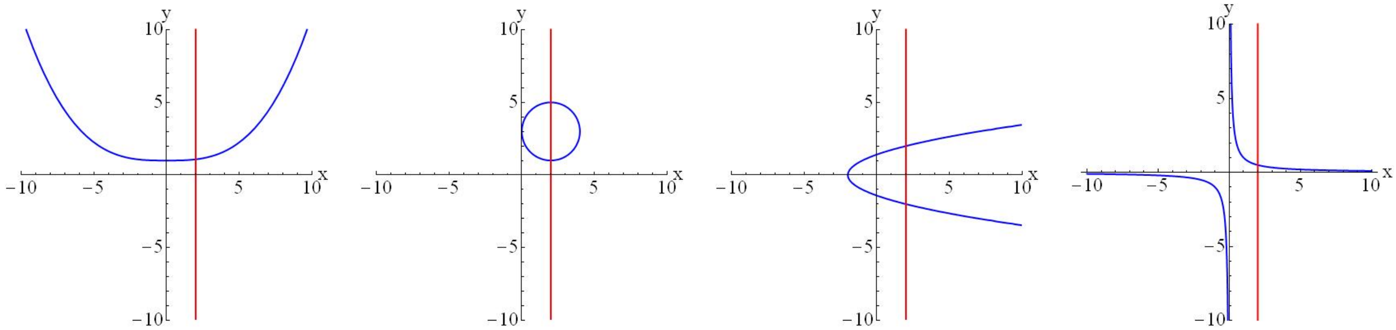


(a) (b) (c) (d)

7. Can you sketch a graph of a function that intersects its own vertical asymptote? If the answer is yes, draw a sketch of such a situation.

Solutions

1. Using the vertical line test, we see only (a) and (d) represent functions.



(a) (b) (c) (d)

2. We must work out $f(-x)$ and see if it equals $f(x)$ (even), $-f(x)$ (odd), or neither (neither)!

$$\begin{aligned} f(-x) &= \frac{12(-x)}{1-(-x)^2} \\ &= \frac{-12x}{1-x^2} \\ &= -\frac{12x}{1-x^2} \\ &= -f(x) \end{aligned}$$

So $f(x) = \dfrac{12}{1-x^3}$ odd.

3. $f(-x) = (-x)^4 + (-x) = x^4 - x$. So $f(x) = \dfrac{12}{1-x^3}$ neither even nor odd.

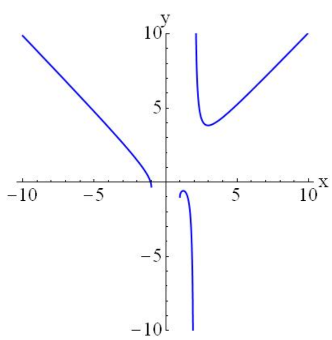
4. At the moment, there are two things to look for when determining domain: division by zero, and square roots.

Division by zero occurs when $x - 2 = 0 \Rightarrow x = 2$. So, $x = 2$ is not in the domain.

The quantity we are taking the square root of must be greater than or equal to zero:
$x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \leq -1$ or $x \geq 1$.

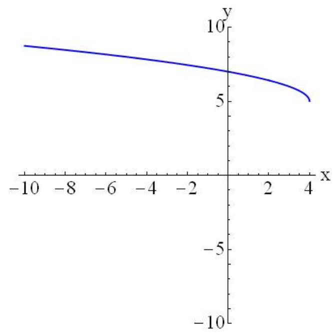
So, the domain requires $x \leq -1$ or $x \geq 1$ and $x \neq 2$. We can write this in a more compact form using interval notation: $x \in (-\infty, -1] \cup [1, 2) \cup (2, \infty)$.

Use a computer to get a sketch to verify:



5. Range is a bit tricky, since we have to think about what comes out of the function. The output of $\sqrt{4-x}$ is $[0,\infty)$. If we add five to that, we get for the range $y \in [5,\infty)$.

Use a computer to get a sketch to verify:



6. The way to do these is to determine the properties of each function, and then look for those properties in the sketches. For the moment, the properties we can easily check are asymptotes and end behaviour (that list will grow as we gain experience with functions).

$y = \dfrac{x^2+2}{x-2}$:

Vertical asymptote when $x-2=0 \Rightarrow x=2$.

End behaviour: $y = \dfrac{x^2+2}{x-2} \sim \dfrac{x^2}{x} = x$ when x is large. Slant asymptote $y=x$.

This looks like sketch (d).

$y = \dfrac{x+4}{4+x^2}$:

Vertical asymptote when $4+x^2=0 \Rightarrow x^2=-4$. No solutions, so no vertical asymptotes.

End behaviour: $y = \dfrac{x+4}{4+x^2} \sim \dfrac{x}{x^2} = \dfrac{1}{x} \sim 0$ when x is large. Horizontal asymptote $y=0$.

This looks like sketch (c).

$y = \dfrac{x+4}{4-x^2}$:

Vertical asymptote when $4-x^2=0 \Rightarrow x^2=4 \Rightarrow x=\pm 2$.

End behaviour: $y = \dfrac{x+4}{4-x^2} \sim \dfrac{x}{-x^2} = -\dfrac{1}{x} \sim 0$ when x is large. Horizontal asymptote $y=0$.

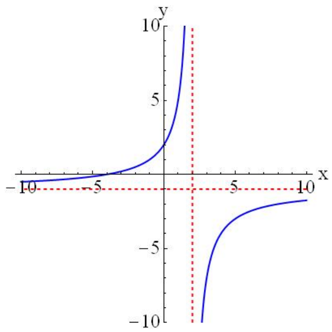
This looks like sketch (b).

$y = \dfrac{x+4}{2-x}$:

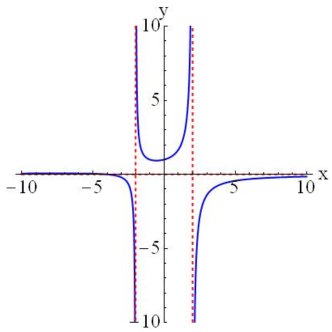
Vertical asymptote when $2-x=0 \Rightarrow x=2$.

End behaviour: $y = \dfrac{x+4}{2-x} \sim \dfrac{x}{-x} = -1$ when x is large. Horizontal asymptote $y=-1$.

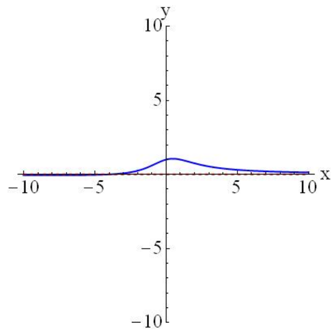
This looks like sketch (a).



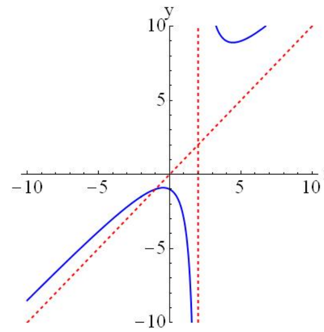
(a) (iv)



(b) (iii)



(c) (ii)



(d) (i)

7. The answer is yes, you can create a function that intersects its own vertical asymptote–the key is to think of a piecewise defined function.

Here is an example, with sketch:

$$f(x) = \begin{cases} 1/x & x < 0 \\ x+1 & x \geq 0 \end{cases}$$