Questions

1. Evaluate without using a calculator; use identities rather than reference triangles. Find $\sec\theta$ and $\csc\theta$ if $\tan\theta = 3$ and $\cos\theta > 0$.

2. Find all solutions to the equation $2\sin^2 x = 1$ in the interval $[0, 2\pi)$.

3. Find all possible solutions to $3\sin t = 2\cos^2 t$.

Solutions

1. Evaluate without using a calculator; use identities rather than reference triangles. Find $\sec\theta$ and $\csc\theta$ if $\tan\theta = 3$ and $\cos\theta > 0$.

First, we need to figure out which Quadrant θ lies in:
$\tan\theta > 0$ means we are in Quadrant I or III.
$\cos\theta > 0$ means we are in Quadrant I or IV.
Therefore, we are in Quadrant I.

The $\sec\theta > 0$ and $\csc\theta > 0$ in Quadrant I.

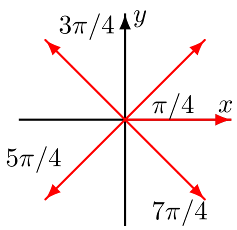
$$\begin{aligned}
\sec\theta &= \sqrt{1+\tan^2\theta} \quad (\text{choose } +\sqrt{\ } \text{ since } \sec\theta > 0) \\
&= \sqrt{1+3^2} \\
&= \sqrt{1+9} = \sqrt{10}
\end{aligned}$$

$$\begin{aligned}
\csc\theta &= \sqrt{\cot^2\theta+1} \quad (\text{choose } +\sqrt{\ } \text{ since } \csc\theta > 0) \\
&= \sqrt{\frac{1}{\tan^2\theta}+1} \\
&= \sqrt{\frac{1}{3^2}+1} \\
&= \sqrt{\frac{1}{9}+1} = \sqrt{\frac{10}{9}} = \frac{\sqrt{10}}{3}
\end{aligned}$$

2. Find all solutions to the equation $2\sin^2 x = 1$ in the interval $[0, 2\pi)$.

$$\begin{aligned}
2\sin^2 x &= 1 \\
\sin^2 x &= \frac{1}{2} \\
\sin x &= \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}
\end{aligned}$$

The angles around the unit circle corresponding to points with y coordinate equal to $+\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$ are the following:



The equation has solution $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ in the interval $[0, 2\pi)$.

3. Find all possible solutions to $3\sin t = 2\cos^2 t$.

$$\begin{aligned} 3\sin t &= 2\cos^2 t \\ 3\sin t &= 2(1-\sin^2 t) \\ 3\sin t &= 2-2\sin^2 t \\ 2\sin^2 t + 3\sin t - 2 &= 0 \end{aligned}$$

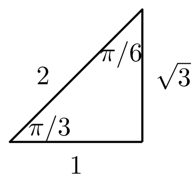
Now, let $y = \sin t$. Then we get

$$\begin{aligned} 2y^2+3y-2 &= 0 \\ y &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-3\pm\sqrt{(-3)^2-4(2)(-2)}}{2(2)} \\ &= \frac{-3\pm 5}{4} \\ &= \frac{-3+5}{4} \text{ or } \frac{-3-5}{4} \\ &= \frac{2}{4} \text{ or } \frac{-8}{4} \\ &= \frac{1}{2} \text{ or } -2 \end{aligned}$$

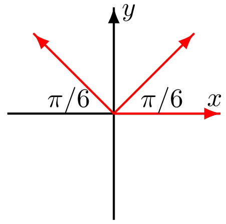
We can exclude the solution $y = -2 = \sin t$ since it is not possible to have the sine of a real number which is less than -1.

Therefore, we now must solve $y = \sin t = \dfrac{1}{2}$ for t.

From a reference triangle, we see the angle is $t = \pi/6$, since $\sin(\pi/6) = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$.



We need to get all the possible solutions, so our work is not done. The sine is positive in Quadrant I and II.



The solution in Quadrant I is $t = \dfrac{\pi}{6} + 2k\pi,\ k = 0, \pm1, \pm2, \ldots$.

The solution in Quadrant II is $t = \pi - \dfrac{\pi}{6} + 2k\pi = \dfrac{5\pi}{6} + 2k\pi, k = 0, \pm1, \pm2, \ldots$.