Questions

1. Solve for x when $\frac{2}{3}x = \frac{1}{15}x + \frac{3}{5}$.

2. Solve for x when $\frac{x}{2} + \frac{x}{5} = \frac{7}{10}$.

3. Solve for x when $20 - \frac{1}{3}x = \frac{1}{2}x$.

4. Is 4 a solution to $\frac{1}{2}(y-2) + 2 = \frac{3}{8}(3y-4)$?

5. Solve for x when $\frac{4}{5}x - \frac{2}{3} = \frac{3x+1}{2}$.

6. Solve for x when $-1 + 5(x-2) = 12x + 3 - 7x$.

7. Solve for x when $9(x+3) - 6 = 24 - 2x - 3 + 11x$.

8. Sketch $y = -2x + 1$. Find the value of y when $x = 0$, $x = -2$, and $x = 1$.

9. Sketch $y = 2x - 5$. Find the value of y when $x = 0$, $x = 2$, and $x = 4$.

10. Sketch $y = 3x + 2$. Find the value of y when $x = -1$, $x = 0$, and $x = 1$.

11. Sketch $4x + 3y = 12$.

12. Sketch $3x + 2y = 6$.

13. Sketch $y = 6 - 2x$.

14. Sketch $x - 6 = 2y$.

15. Sketch $y - 2 = 3y$.

16. Sketch $2x + 9 = 5x$.

17. Sketch $2x + 5y - 2 = -12$.

18. Find the slope of the straight line that passes through the points $(4, 1)$ and $(6, 7)$.

19. Find the slope of the straight line that passes through the points $(11, 2)$ and $(5, 14)$.

20. Find the slope of the straight line that passes through the points $(-6, -5)$ and $(2, -7)$.

21. Write the equation for a straight line in slope-intercept form with slope $m = \frac{2}{3}$ and y-intercept $(0, 5)$.

22. Write the equation for a straight line in slope-intercept form with slope $m = 5$ and y-intercept $(0, -6)$.

23. Write the equation for a straight line in slope-intercept form with slope $m = \frac{2}{3}$ and y-intercept $(0, 1/2)$.

24. Sketch the straight line $y = mx + b$ where $m = \frac{1}{3}$ and $b = -2$.

25. Sketch the straight line $y = mx + b$ where $m = -\frac{3}{2}$ and $b = 4$.

26. Sketch the straight line $y = 3x$.

27. A line has a slope of $\frac{11}{4}$. What is the slope of a line parallel to it? What is the slope of a line perpendicular to it?

28. A line has equation $y = \frac{3}{5}x - 5$. What is the slope of a line parallel to it? What is the slope of a line perpendicular to it?

29. During the years from 1980 to 2005 the total income for the U.S. federal budget can be approximated by the equation $y = 14(4x + 35)$, where x is the number of years since 1980 and y is the amount of money in billions of dollars (source: U.S. Office of Management and Budget).

Write the equation in slope-intercept form. Find the slope and y-intercept. What is the meaning of the slope in this situation?

30. Find the equation of the line that passes through the point $(5, -3)$ and has slope $m = -\frac{2}{5}$.

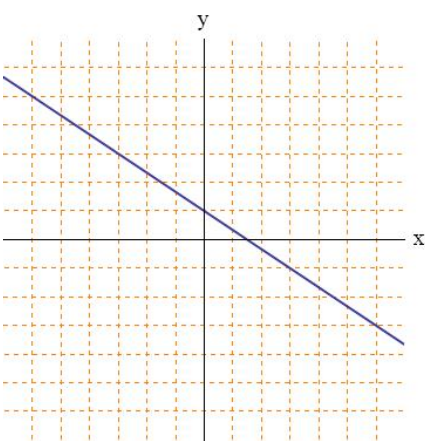
31. Find the equation of the line that passes through the points $\left(1, \frac{5}{6}\right)$ and $\left(3, \frac{3}{2}\right)$.

32. Find the equation of the line that passes through the points $(2, 0)$ and $\left(\frac{3}{2}, \frac{1}{2}\right)$.

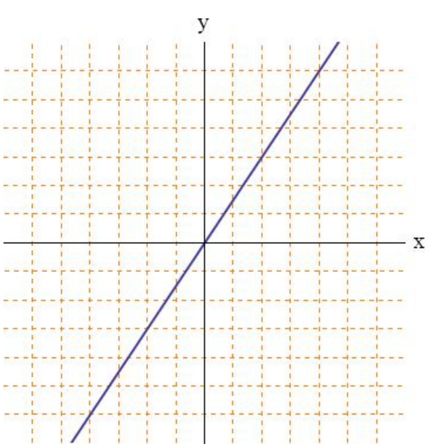
33. Find the equation of the line that passes through the point $(4, 3)$ and has slope $m = -2$.

34. Find the equation of the line that passes through the points $(1, -8)$ and $(2, -14)$.

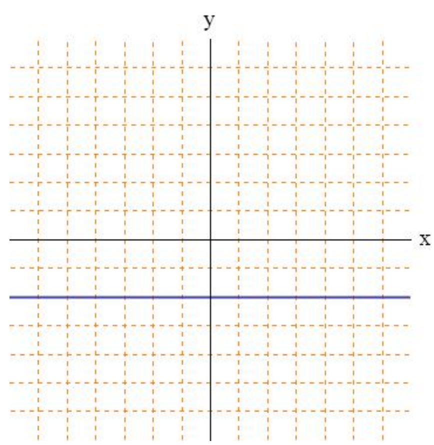
35. Write the equation of the line given below.



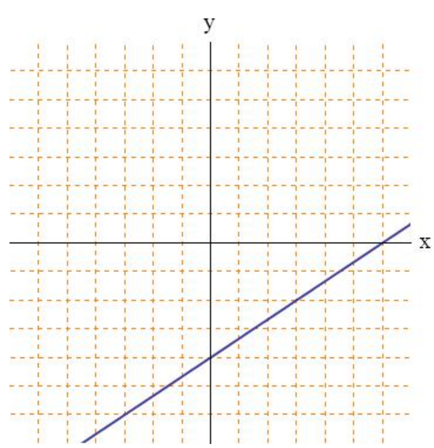
36. Write the equation of the line given below.



37. Write the equation of the line given below.



38. Write the equation of the line given below.



Solutions

1. The LCD (lowest common denominator) is 15, so multiply the equation by 15 to remove the fractions.

$$\begin{aligned}
\frac{2}{3}x &= \frac{1}{15}x + \frac{3}{5} \\
15 \cdot \left(\frac{2}{3}x\right) &= 15 \cdot \left(\frac{1}{15}x + \frac{3}{5}\right) \\
10x &= 15 \cdot \frac{1}{15}x + 15 \cdot \frac{3}{5} \text{ distribute!} \\
10x &= x + 9 \text{ simplify} \\
10x - x &= x + 9 - x \text{ addition principle} \\
9x &= 9 \text{ simplify} \\
\frac{1}{9} \cdot 9x &= \frac{1}{9} \cdot 9 \text{ multiplication principle} \\
x &= 1 \text{ simplify}
\end{aligned}$$

2. LCD is 10.

$$\begin{aligned}
\frac{x}{2} + \frac{x}{5} &= \frac{7}{10} \\
10 \cdot \left(\frac{x}{2} + \frac{x}{5}\right) &= 10 \cdot \frac{7}{10} \\
10 \cdot \frac{x}{2} + 10 \cdot \frac{x}{5} &= 7 \\
5x + 2x &= 7 \\
7x &= 7 \\
\frac{1}{7} \cdot 7x &= \frac{1}{7} \cdot 7 \\
x &= 1
\end{aligned}$$

3. LCD is 6.

$$\begin{aligned}
20 - \frac{1}{3}x &= \frac{1}{2}x \\
6 \cdot \left(20 - \frac{1}{3}x\right) &= 6 \cdot \frac{1}{2}x \\
6 \cdot 20 - 6 \cdot \frac{1}{3}x &= 3x \\
120 - 2x &= 3x \\
120 - 2x + 2x &= 3x + 2x \\
120 &= 5x \\
\frac{1}{5} \cdot 120 &= \frac{1}{5} \cdot 5x \\
24 &= x
\end{aligned}$$

4. You could substitute $y = 4$ to check, but I am going to solve it instead. LCD is 8.

$$\begin{aligned}
\frac{1}{2}(y-2)+2 &= \frac{3}{8}(3y-4) \\
8\cdot\left(\frac{1}{2}(y-2)+2\right) &= 8\cdot\frac{3}{8}(3y-4) \\
8\cdot\frac{1}{2}(y-2)+8\cdot 2 &= 3(3y-4) \\
4(y-2)+16 &= 9y-12 \\
4y-8+16 &= 9y-12 \\
4y+8 &= 9y-12 \\
4y+8-9y-8 &= 9y-12-9y-8 \\
-5y &= -20 \\
\frac{1}{-5}\cdot(-5y) &= \frac{1}{-5}\cdot(-20) \\
y &= 4
\end{aligned}$$

5. LCD is 30.

$$\begin{aligned}
\frac{4}{5}x-\frac{2}{3} &= \frac{3x+1}{2} \\
30\cdot\left(\frac{4}{5}x-\frac{2}{3}\right) &= 30\cdot\frac{3x+1}{2} \\
30\cdot\frac{4}{5}x-30\cdot\frac{2}{3} &= 30\cdot\frac{1}{2}\cdot(3x+1)
\end{aligned}$$

Note in above I wrote $\dfrac{3x+1}{2}$ as $\frac{1}{2}\cdot(3x+1)$. Doing this helps reduce errors!

$$\begin{aligned}
24x-20 &= 15\cdot(3x+1) \\
24x-20 &= 45x+15 \\
24x-20-45x+20 &= 45x+15-45x+20 \\
-21x &= 35 \\
\frac{1}{-21}\cdot(-21x) &= \frac{1}{-21}\cdot 35 \\
x &= -\frac{35}{21} = -\frac{5}{3}
\end{aligned}$$

6.

$$\begin{aligned}
-1+5(x-2) &= 12x+3-7x \\
-1+5x-10 &= 5x+3 \\
5x-9-5x &= 5x+3-5x \\
-9 &= 3
\end{aligned}$$

We have to <u>interpret</u> what we have found. Since -9 never equals 3, the equation is never true no matter what value of x we put in. This means the equation has no solution.

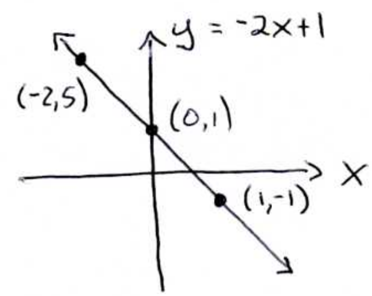
7.

$$\begin{aligned} 9(x+3)-6 &= 24-2x-3+11x \\ 9x+27-6 &= 21+9x \\ 9x+21 &= 21+9x \\ 9x+21-9x &= 21+9x-9x \\ 21 &= 21 \end{aligned}$$

We have to interpret what we have found. Since 21 is always equal to 21, the equation is true for any value of x that we try. Therefore, there are an infinite number of solutions.

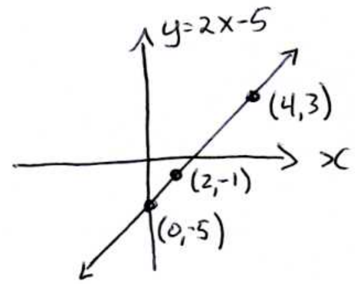
8. $y = -2x + 1$

When $x = 0 \Rightarrow y = -2(0) + 1 = 1,$ so the ordered pair is $(0, 1)$.
When $x = -2 \Rightarrow y = -2(-2) + 1 = 5,$ so the ordered pair is $(-2, 5)$.
When $x = 1 \Rightarrow y = -2(1) + 1 = -1,$ so the ordered pair is $(1, -1)$.



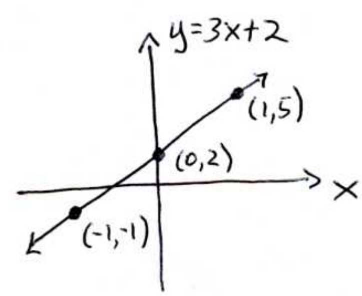
9. $y = 2x - 5$

When $x = 0 \Rightarrow y = 2(0) - 5 = -5,$ so the ordered pair is $(0, -5)$.
When $x = 2 \Rightarrow y = 2(2) - 5 = -1,$ so the ordered pair is $(2, -1)$.
When $x = 4 \Rightarrow y = 2(4) - 5 = 3,$ so the ordered pair is $(4, 3)$.



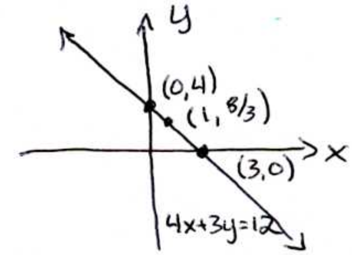
10. $y = 3x + 2$

When $x = -1 \Rightarrow y = 3(-1) + 2 = -1,$ so the ordered pair is $(-1, -1)$.
When $x = 0 \Rightarrow y = 3(0) + 2 = 2,$ so the ordered pair is $(0, 2)$.
When $x = 1 \Rightarrow y = 3(1) + 2 = 5,$ so the ordered pair is $(1, 5)$.



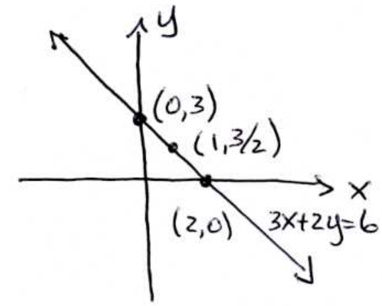
11. $4x + 3y = 12$

When $x = 0 \Rightarrow 4(0) + 3y = 12 \Rightarrow y = 4$ so the ordered pair is $(0, 4)$.
When $y = 0 \Rightarrow 4x + 3(0) = 12 \Rightarrow x = 3$ so the ordered pair is $(3, 0)$.
When $x = 1 \Rightarrow 4(1) + 3y = 12 \Rightarrow y = 8/3$ so the ordered pair is $(1, 8/3)$.



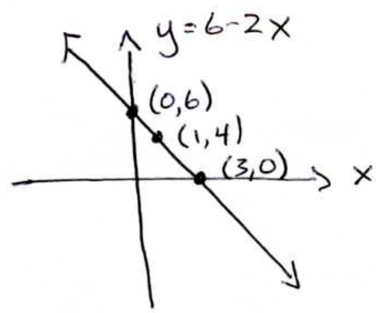
12. $3x + 2y = 6$

When $x = 0 \Rightarrow 3(0) + 2y = 6 \Rightarrow y = 3$ so the ordered pair is $(0, 3)$.
When $y = 0 \Rightarrow 3x + 2(0) = 6 \Rightarrow x = 2$ so the ordered pair is $(2, 0)$.
When $x = 1 \Rightarrow 3(1) + 2y = 6 \Rightarrow y = 3/2$ so the ordered pair is $(1, 3/2)$.



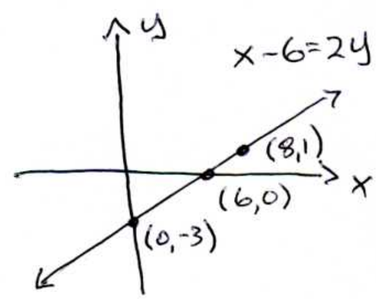
13. $y = 6 - 2x$

When $x = 0 \Rightarrow y = 6 - 2(0) \Rightarrow y = 6$ so the ordered pair is $(0, 6)$.
When $y = 0 \Rightarrow (0) = 6 - 2x \Rightarrow x = 3$ so the ordered pair is $(3, 0)$.
When $x = 1 \Rightarrow y = 6 - 2(1) \Rightarrow y = 4$ so the ordered pair is $(1, 4)$.

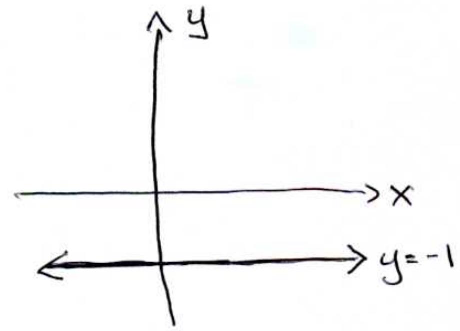


14. $x - 6 = 2y$

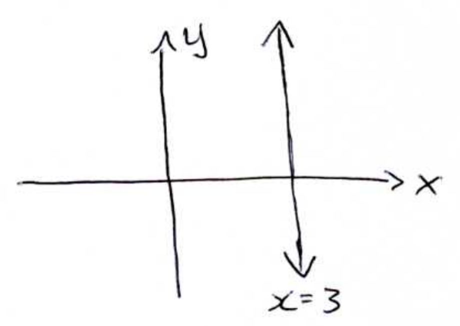
When $x = 0 \Rightarrow (0) - 6 = 2y \Rightarrow y = -3$ so the ordered pair is $(0, -3)$.
When $y = 0 \Rightarrow x - 6 = 2(0) \Rightarrow x = 6$ so the ordered pair is $(6, 0)$.
When $x = 8 \Rightarrow (8) - 6 = 2y \Rightarrow y = 1$ so the ordered pair is $(8, 1)$.



15. $y - 2 = 3y$. There is no x in the equation. Simplification shows this is a horizontal line, $y = -1$.

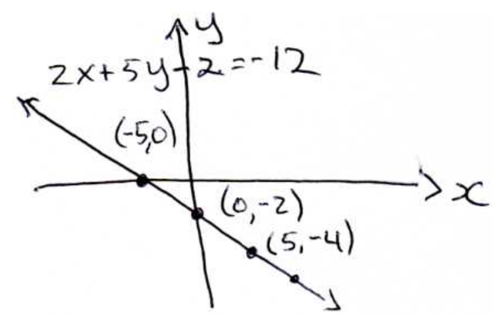


16. $2x + 9 = 5x$. There is no y in the equation. Simplification shows this is a vertical line, $x = 3$.



17. $2x + 5y - 2 = -12 \Rightarrow 2x + 5y = -10$

When $x = 0 \Rightarrow 2(0) + 5y = -10 \Rightarrow y = -2$ so the ordered pair is $(0, -2)$.
When $y = 0 \Rightarrow 2x + 5(0) = -10 \Rightarrow x = -5$ so the ordered pair is $(-5, 0)$.
When $x = 5 \Rightarrow 2(5) + 5y = -10 \Rightarrow y = -4$ so the ordered pair is $(5, -4)$.



18. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1-7}{4-6} = \dfrac{-6}{-2} = 3.$

19. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{2-14}{11-5} = \dfrac{-12}{6} = -2.$

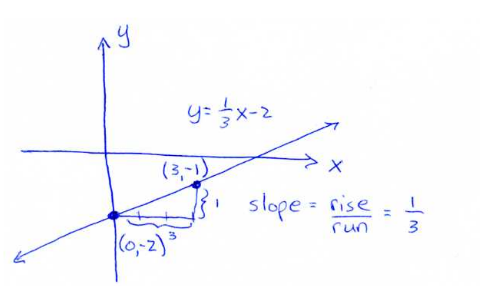
20. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{-5-(-7)}{-6-2} = \dfrac{2}{-8} = -\dfrac{1}{4}.$

21. $y = \dfrac{2}{3}x + 5.$

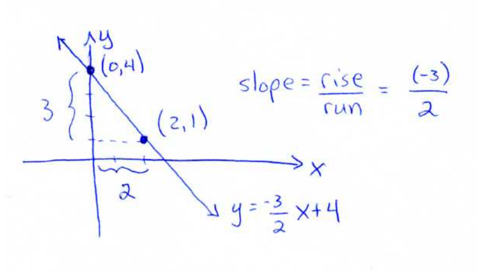
22. $y = 5x - 6.$

23. $y = \dfrac{2}{3}x + \dfrac{1}{2}.$

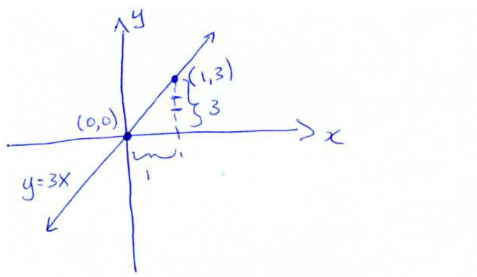
24. $y = \dfrac{1}{3}x - 2.$



25. $y = -\frac{3}{2}x + 4.$



26. $y = 3x.$
Intercept is $b = 0$. Slope is $m = \frac{\text{rise}}{\text{run}} = \frac{3}{1}.$



27. Parallel: $\frac{11}{4}$. Perpendicular: $-\frac{4}{11}$.

28. Parallel: $\frac{3}{5}$. Perpendicular: $-\frac{5}{3}$.

29. $y = 14(4x + 35) = 56x + 490 \Rightarrow$ slope $= 56$ and y-intercept is $(0, 490)$.

The slope is the amount of increase in income of the federal budget in billions of dollars per year.

Aside: This equation is not as good as it could be, since x represents the number of years since 1980. The equation would be improved if the independent variable represented the year. We can make this change by introducing a change in variables.

Let z be the year. Then $z = 1980 + x$. Therefore, $x = z - 1980$. The equation becomes

$$y = 56x + 490$$
$$y = 56(z - 1980) + 490 = 56z - 110,390$$

What was the federal budget in 1987? Answer: $y = 56z - 110,390 = 56(1987) - 110,390 = 882$ billion dollars. This is the same answer you get if you use $y = 56x + 490$ with $x = 7$.

30. Use the slope-point equation of a line.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-3) &= -\frac{2}{5}(x - 5) \\ y + 3 &= -\frac{2}{5}x + 2 \\ y &= -\frac{2}{5}x - 1 \end{aligned}$$

31. slope $= \frac{\Delta y}{\Delta x} = \frac{\frac{5}{6} - \frac{3}{2}}{1 - 3} = \frac{\left(-\frac{4}{6}\right)}{-2} = \frac{1}{-2} \cdot \left(-\frac{4}{6}\right) = \frac{1}{3}.$

Now use the slope-point equation of a line.

$$\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - \frac{5}{6} &= \frac{1}{3}(x-1) \\
y - \frac{5}{6} &= \frac{1}{3}x - \frac{1}{3} \\
y &= \frac{1}{3}x - \frac{1}{3} + \frac{5}{6} \\
y &= \frac{1}{3}x - \frac{2}{6} + \frac{5}{6} \\
y &= \frac{1}{3}x + \frac{3}{6} \\
y &= \frac{1}{3}x + \frac{1}{2}
\end{aligned}$$

32. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{0 - \frac{1}{2}}{2 - \frac{3}{2}} = \dfrac{\left(-\frac{1}{2}\right)}{\left(\frac{1}{2}\right)} = \dfrac{2}{1} \cdot \left(-\dfrac{1}{2}\right) = -1.$

Now use the slope-point equation of a line.

$$\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 0 &= -1(x-2) \\
y &= -x + 2
\end{aligned}$$

33. Use the slope-point equation of a line.

$$\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - (3) &= -2(x-4) \\
y - 3 &= -2x + 8 \\
y &= -2x + 8 + 3 \\
y &= -2x + 11
\end{aligned}$$

34. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{-8 - (-14)}{1 - 2} = \dfrac{-8 + 14}{-1} = \dfrac{6}{-1} = -6.$

Now use the slope-point equation of a line.

$$\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - (-8) &= -6(x-1) \\
y + 8 &= -6x + 6 \\
y &= -6x + 6 - 8 \\
y &= -6x - 2
\end{aligned}$$

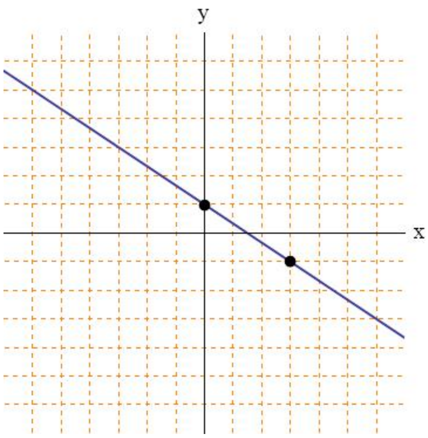
35. You need to be able to read these off the sketch. Look for two points that the line crosses a grid line intersection. Two points: $(0, 1)$ and $(3, -1)$.

Rise $= -2$, Run $= 3$.

slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{-2}{3} = -\dfrac{2}{3}$.

y-intercept $b = 1$.

$y = mx + b \Rightarrow y = -\dfrac{2}{3}x + 1$.



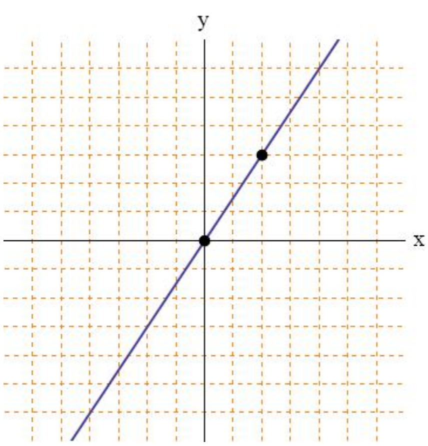
36. Two points: $(0, 0)$ and $(2, 3)$.

Rise $= 3$, Run $= 2$.

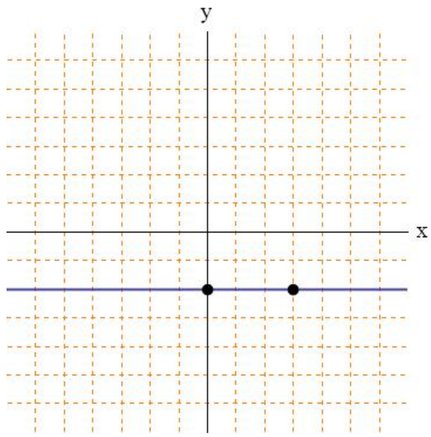
slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{3}{2}$.

y-intercept $b = 0$.

$y = mx + b \Rightarrow y = \dfrac{3}{2}x$.



37. This is a horizontal line, so it's equation is just $y = -2$.



38. Two points: $(0, -4)$ and $(3, -2)$.

Rise $= 2$, Run $= 3$.

slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{2}{3}$.

y-intercept $b = -4$.

$y = mx + b \Rightarrow y = \dfrac{2}{3}x - 4$.