Questions

1. Find the vertex and axis of symmetry of $f(x) = 5x^2 - 6x + 4$.

2. Use completing the square to describe and sketch the graph of $f(x) = 5x^2 - 25x + 12$.

3. Find the point of intersection between the line that passes through the two points $(1, 2)$ and $(3, 7)$, and the line perpendicular to this first line that passes through the point $(2, 4)$. Draw a sketch of the situation that is as accurate as possible.

Solutions

1. Find the vertex and axis of symmetry of $f(x) = 5x^2 - 6x + 4$.

I'll use completing the square to write this in vertex form, and then just read off the vertex and axis of symmetry.

$$\begin{aligned}
f(x) &= 5x^2 - 6x + 4 \\
&= 5\left(x^2 + \left(\frac{-6}{5}\right)x\right) + 4 \\
&= 5\left(x^2 + \left(\frac{-6}{5}\right)x + \left(\frac{-3}{5}\right)^2 - \left(\frac{-3}{5}\right)^2\right) + 4 \\
&= 5\left(x^2 + \left(\frac{-6}{5}\right)x + \left(\frac{-3}{5}\right)^2 - \left(\frac{-3}{5}\right)^2\right) + 4 \\
&= 5\left(\left[x + \left(\frac{-3}{5}\right)\right]^2 - \left(\frac{-3}{5}\right)^2\right) + 4 \\
&= 5\left(\left[x - \frac{3}{5}\right]^2 - \left(\frac{-3}{5}\right)^2\right) + 4 \qquad \text{notice how our treatment of the minus sign for } b \text{ was done!} \\
&= 5\left[x - \frac{3}{5}\right]^2 - 5\left(\frac{-3}{5}\right)^2 + 4 \qquad \text{simplify} \\
&= 5\left[x - \frac{3}{5}\right]^2 - \frac{9}{5} + \frac{20}{5} \\
&= 5\left[x - \frac{3}{5}\right]^2 + \frac{11}{5}
\end{aligned}$$

This is now in the vertex form, and we can identify the vertex as $(h, k) = \left(\frac{3}{5}, \frac{11}{5}\right)$ and axis of symmetry $x = \frac{3}{5}$.

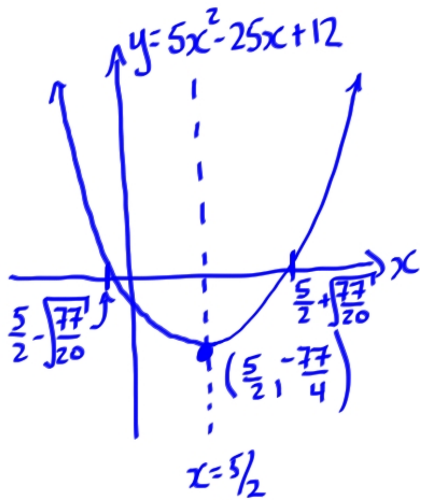
2. Use completing the square to describe and sketch the graph of $f(x) = 5x^2 - 25x + 12$.

$$\begin{aligned}
f(x) &= 5x^2 - 25x + 12 \\
&= 5\left(x^2 + (-5)x\right) + 12 \\
&= 5\left(x^2 + (-5)x + \left(\frac{-5}{2}\right)^2 - \left(\frac{-5}{2}\right)^2\right) + 12 \\
&= 5\left(x^2 + (-5)x + \left(\frac{-5}{2}\right)^2 - \left(\frac{-5}{2}\right)^2\right) + 12 \\
&= 5\left(\left[x + \left(\frac{-5}{2}\right)\right]^2 - \left(\frac{-5}{2}\right)^2\right) + 12 \\
&= 5\left(\left[x - \frac{5}{2}\right]^2 - \left(\frac{-5}{2}\right)^2\right) + 12 \qquad \text{notice how our treatment of the minus sign for } b \text{ was done!}
\end{aligned}$$

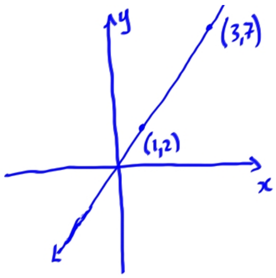
$$\begin{aligned}
&= 5\left[x-\frac{5}{2}\right]^2 - 5\left(\frac{-5}{2}\right)^2 + 12 \qquad \text{simplify}\\
&= 5\left[x-\frac{5}{2}\right]^2 - \frac{125}{4} + \frac{48}{4}\\
&= 5\left[x-\frac{5}{2}\right]^2 - \frac{77}{4}
\end{aligned}$$

This is now in the vertex form, and we can identify the vertex as $(h,k) = \left(\frac{5}{2}, -\frac{77}{4}\right)$ and axis of symmetry $x = \frac{5}{2}$. It will open upwards since the coefficient in front of x^2 is positive. Since the vertex is below the x axis and the graph opens up, the graph will intersect the x-axis in two places. These roots can be found by setting $g(x) = 0$:

$$\begin{aligned}
g(x) = 0 &= 5\left[x-\frac{5}{2}\right]^2 - \frac{77}{4}\\
5\left[x-\frac{5}{2}\right]^2 &= \frac{77}{4}\\
\left[x-\frac{5}{2}\right]^2 &= \frac{77}{20}\\
x-\frac{5}{2} &= \pm\sqrt{\frac{77}{20}}\\
x &= \frac{5}{2} \pm \sqrt{\frac{77}{20}}
\end{aligned}$$



3. Find the point of intersection between the line that passes through the two points $(1,2)$ and $(3,7)$, and the line perpendicular to this first line that passes through the point $(2,4)$. Draw a sketch of the situation that is as accurate as possible.



$\hookrightarrow$ slope $= \frac{\Delta y}{\Delta x} = \frac{7-2}{3-1} = \frac{5}{2}$

$y - y_1 = m(x - x_1)$ use $\begin{cases} m = 5/2 \\ x_1 = 1 \\ y_1 = 2 \end{cases}$

$y - 2 = \frac{5}{2}(x-1)$

$y = \frac{5}{2}x - \frac{5}{2} + 2$

$y = \frac{5}{2}x - \frac{1}{2}$.

A line perpendicular to this will have slope $-2/5$. If it passes through point $(2,4)$, it will have equation

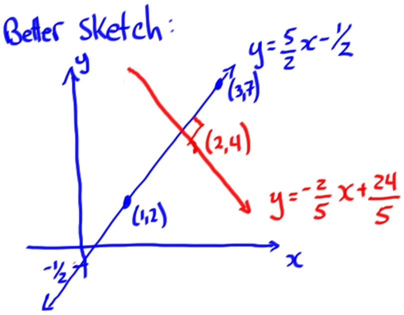
$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{2}{5}(x-2)$

$y = -\frac{2}{5}x + \frac{4}{5} + 4$

$y = -\frac{2}{5}x + \frac{24}{5}$

A Better Sketch:



The lines intersect where

$y = -\frac{2}{5}x + \frac{24}{5} = \frac{5}{2}x - \frac{1}{2}$

solve for x.

$\frac{24}{5} + \frac{1}{2} = \frac{5}{2}x + \frac{2}{5}x$

$10 \cdot \frac{48+5}{10} = \frac{25x+4x}{10} \cdot 10$

$53 = 29x \Rightarrow x = \frac{53}{29}$

Get y from either of the equations. Pick $y = \frac{5}{2}x - \frac{1}{2}$

$= \frac{5}{2}\left(\frac{53}{29}\right) - \frac{1}{2} = \frac{118}{29}$.

So the point of intersection is $\left(\frac{53}{29}, \frac{118}{29}\right)$.