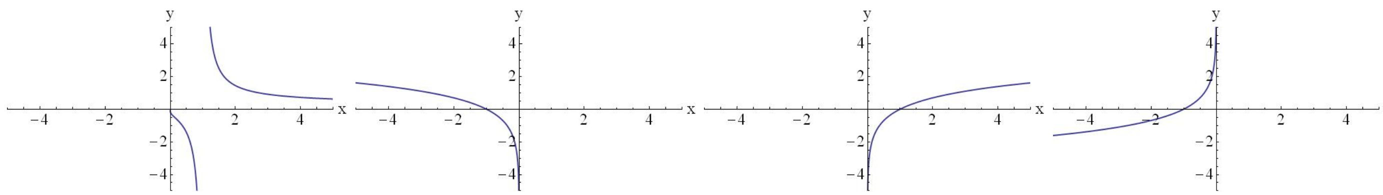
Questions

1. Find the inverse function $f^{-1}(x)$ if $f(x) = e^{-3x} + 2$. Verify you have the correct answer by checking that $f(f^{-1}(x)) = x$.

2. Given $f(x) = \ln(\sqrt{x})$, $g(x) = e^{x/4}$, and $h(x) = x^2$. Find the composition $(h \circ g \circ f)(x)$ and simplify as much as possible. Your final answers should **not** have exponentials and logarithms in them.

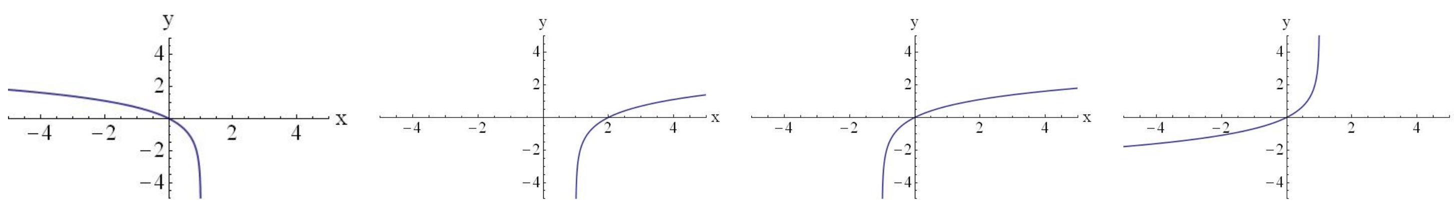
3. Match the function with its graph

(a) $y = \ln(-x)$ (b) $y = \ln(x)$ (c) $y = -\ln(-x)$ (d) $y = (\ln x)^{-1}$



4. Match the function with its graph

(a) $y = \ln(x+1)$ (b) $y = \ln(x-1)$ (c) $y = -\ln(-x+1)$ (d) $y = \ln(1-x)$



Solutions

1. Find the inverse function $f^{-1}(x)$ if $f(x) = e^{-3x} + 2$. Verify you have the correct answer by checking that $f(f^{-1}(x)) = x$.

$$
\begin{aligned}
f(x) &= e^{-3x} + 2 \\
y &= e^{-3x} + 2 \\
x &= e^{-3y} + 2 \text{ interchange } x \text{ and } y \\
x - 2 &= e^{-3y} \text{ solve for } y \\
\ln(x-2) &= \ln e^{-3y} \text{ solve for } y \\
\ln(x-2) &= -3y \text{ simplify using logarithm rules} \\
y &= -\frac{1}{3}\ln(x-2)
\end{aligned}
$$

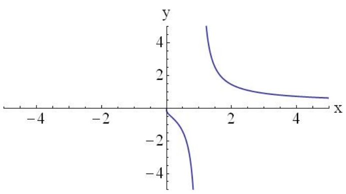
$$
\begin{aligned}
f(f^{-1}(x)) &= f\Big(-\frac{1}{3}\ln(x-2)\Big) \\
&= \exp(-3(-\frac{1}{3}\ln(x-2))) + 2 \\
&= \exp(\ln(x-2)) + 2 \\
&= x - 2 + 2 = x
\end{aligned}
$$

2. Given $f(x) = \ln(\sqrt{x})$, $g(x) = e^{x/4}$, and $h(x) = x^2$. Find the composition $(h \circ g \circ f)(x)$ and simplify as much as possible. Your final answers should **not** have exponentials and logarithms in them.

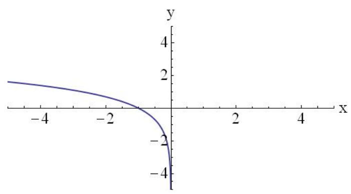
$$
\begin{aligned}
(h \circ g \circ f)(x) &= h\big(g(f(x))\big) \\
&= h\big(g(\ln(x^{1/2}))\big) \\
&= h\Big(\exp(\frac{\ln(x^{1/2}}{4})\Big) \\
&= h\Big(\exp(\frac{1}{4}\ln(x^{1/2}))\Big) \\
&= h\Big(\exp(\ln((x^{1/2})^{1/4}))\Big) \\
&= h\Big(\exp(\ln(x^{1/8}))\Big) \\
&= h\big(x^{1/8}\big) \\
&= (x^{1/8})^2 \\
&= x^{1/4}
\end{aligned}
$$

3. Match the function with its graph

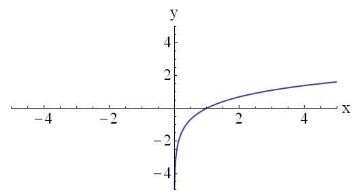
(a) $y = \ln(-x)$ (b) $y = \ln(x)$ (c) $y = -\ln(-x)$ (d) $y = (\ln x)^{-1}$



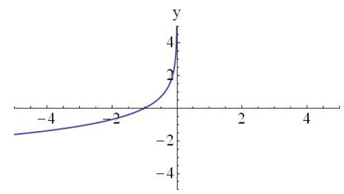
(d)



(a)



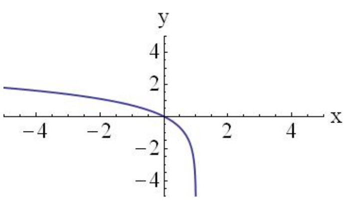
(b)



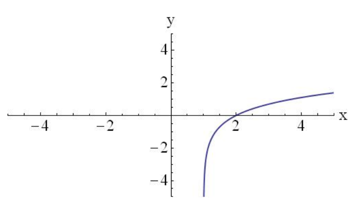
(c)

4. Match the function with its graph

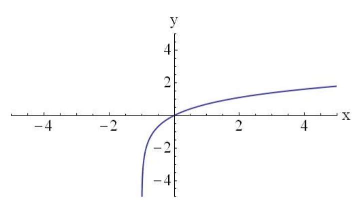
(a) $y = \ln(x+1)$ (b) $y = \ln(x-1)$ (c) $y = -\ln(-x+1)$ (d) $y = \ln(1-x)$



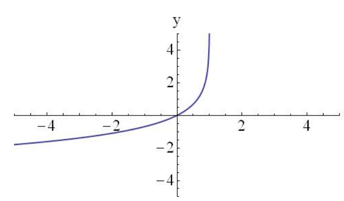
(d)



(b)



(a)



(c)