Note: The following formulas from Mathematics of Finance will be provided on tests:

Simple Interest: $A = P(1 + rt)$

Compound Interest: $A = P(1 + i)^k$

Continuous Interest: $A = Pe^{rt}$

Savings formula: $A = d\left[\dfrac{(1+i)^k - 1}{i}\right]$

Amortization formula: $P = d\left[\dfrac{1-(1+i)^{-k}}{i}\right]$

Questions

1. You put \$98.45 in a savings account which pays simple interest of 6% a year. How much money do you have in the savings account after 4 years?

2. You put \$98.45 in a savings account which pays compound interest of 6% a month. How much money do you have in the savings account after 4 years?

3. \$1000 is deposited at 6% per year. Find the balance at the end of one year, if the interest paid is a) simple interest b) compounded quarterly.

4. You wish to remodel your kitchen, and estimate it will cost \$35,000 to do. If you can afford to save \$500 a month in a savings account that earns 4% annual interest, how long will it take you to save enough to remodel the kitchen?

5. What should your monthly deposit be in a savings account with 7% annual interest compounded monthly if you want to save \$3000 in 24 months for the down payment on a new car?

Solutions

1. You put \$98.45 in a savings account which pays simple interest of 6% a year. How much money do you have in the savings account after 4 years?

Solution Identify $P = \$98.45$, $r = 6\%$ and time period of one year. So for four years, $t = 4$.

$$\begin{aligned} A &= P(1 + rt) \\ &= \$98.45(1 + 0.06 \times 4) \\ &= \$122.08 \end{aligned}$$

2. You put \$98.45 in a savings account which pays compound interest of 6% a month. How much money do you have in the savings account after 4 years?

Solution Identify $P = \$98.45$, $i = 6\%$. So for four years, $t = 4$ and $m = 12$ compounding periods per year, so $n = mt = 12 \times 4 = 36$.

$$A = P(1 + i)^n = \$98.45(1 + 0.06)^{48} = \$1613.98$$

3. \$1000 is deposited at 6% per year. Find the balance at the end of one year, if the interest paid is a) simple interest b) compounded quarterly.

a) The principal is $P =$\$1000, and the nominal rate is $r =$6% $= 0.06$. After one year, $t = 1$. If we use simple interest, we have an accumulated balance of

$$A = P(1 + rt) = \$1000.00(1 + 0.06 \times 1) = \$1060.00$$

b) The nominal annual rate is $r =$6% $= 0.06$, when compounded quarterly, means we have $m = 4$, so $i = r/m = 0.06/4 = 0.015$. One year corresponds to $t = 1$, so after one year we have $n = mt = 4 \times 1 = 4$,

$$A = P(1 + i)^n = \$1000.00(1 + 0.015)^4 = \$1061.36.$$

4. You wish to remodel your kitchen, and estimate it will cost \$35,000 to do. If you can afford to save \$500 a month in a savings account that earns 4% annual interest, how long will it take you to save enough to remodel the kitchen?

Solution We use the savings formula, with $A = \$35,000$, $i = r/m = 0.04/12 = 0.00333333$, and $d = \$500$. We need to figure out n, the number of months it will take to save \$35,000.

$$\begin{aligned} A &= d\left[\frac{(1+i)^n - 1}{i}\right] \\ \$35000 &= \$500\left[\frac{(1 + 0.00333333)^n - 1}{0.00333333}\right] \\ \frac{\$35000 \times 0.00333333}{\$500} &= 1.00333333^n - 1 && \text{first, isolate the } 1.003333^k \\ 0.233333 &= 1.00333333^n - 1 \\ 1 + 0.233333 &= 1.00333333^n \\ 1.233333 &= 1.00333333^n && \text{now we take the natural logarithm} \\ \ln(1.233333) &= \ln(1.00333333^n) \\ \ln(1.233333) &= n\ln(1.00333333) && \text{using our logarithm rule } \ln(b^A) = A\ln(b) \\ n &= \frac{\ln(1.233333)}{\ln(1.00333333)} && \text{solve for } k \\ n &= 63.027 \end{aligned}$$

So it will take 63 months (5 years 3 months) to save for the kitchen remodel.

5. What should your monthly deposit be in a savings account with 7% annual interest compounded monthly if you want to save \$3000 in 24 months for the down payment on a new car?

Solution We use the savings formula, since we know $A = \$3000$, $i = r/m = 0.07/12 = 0.00583333$, and $n = mt = 12 \times 2 = 24$ is the time period. We need to figure out d.

$$\begin{aligned}
A &= d\left[\frac{(1+i)^n - 1}{i}\right] \\
\$3000 &= d\left[\frac{(1+0.00583333)^{24} - 1}{0.00583333}\right] \\
\$3000 &= d\,[25.681] \\
\frac{\$3000}{25.681} &= d \\
\$116.82 &= d
\end{aligned}$$

So you will have to save \$116.82 per month to have a \$3000 down payment in 24 months.