*Note: There will be no trig based questions on the tests.*

**Questions**

**1.** Eliminate the parameter $t$ and identify the graph of the parametric curve $x = t^2$, $y = t + 1$.

**2.** Eliminate the parameter $t$ using trig identities and identify the graph of the parametric curve $x = 4 + 3\cos t$, $y = -5 + 2\sin t$.

**3.** Find a parameterization of the line segment between the points $(1, 2)$ and $(-4, 5)$.

**4.** Sketch the parametric curve

$$x = 1 + t$$
$$y = t$$

by eliminating the parameter.

**5.** Sketch the parametric curve

$$x = 5 - 3t$$
$$y = 2 + t$$
$$-1 \leq t \leq 3$$

by eliminating the parameter.

**6.** Sketch the parametric curve

$$x = t - 3$$
$$y = 2/t$$
$$|t| \leq 5$$

by eliminating the parameter.

**7.** Sketch the parametric curve

$$x = 2t^2 - 1$$
$$y = t^4$$

by eliminating the parameter.

**8.** For the parametric curve

$$x = at + b$$
$$y = ct + d$$

where $a$ and $c$ are not both zero.

(a) Eliminate the parameter $t$ and explain why its graph is a line.
(b) Find the slope, $y$-intercept, and $x$-intercept if they exist.
(c) Under what conditions would the line be horizontal? Vertical?

**9.** For the parametric curve

$$x = tc + (1 - t)a$$
$$y = td + (1 - t)b$$
$$0 \leq t \leq 1$$

(a) Determine the value of $t$ that divides the line into two equal segments.
(b) Determine the value of $t$ that divides the line into three equal segments.
(c) What do you think the values of $t$ should be to split the line into $n$ equal segments?

**Solutions**

**1.** Eliminate the parameter $t$ and identify the graph of the parametric curve $x = t^2$, $y = t + 1$.

We can write $t = y - 1$ and substitute as follows:

$$\begin{aligned} x &= t^2 \\ &= (y-1)^2 \end{aligned}$$

This is a parabola, which opens to the right with vertex $(0, 1)$.

**2.** Eliminate the parameter $t$ using trig identities and identify the graph of the parametric curve $x = 4 + 3\cos t$, $y = -5 + 2\sin t$.

We see this is most likely an ellipse, so try to use $\cos^2 t + \sin^2 t = 1$

$$\cos t = \frac{x-4}{3} \Rightarrow \cos^2 t = \frac{(x-4)^2}{3^2}$$
$$\sin t = \frac{y+5}{2} \Rightarrow \sin^2 t = \frac{(y+5)^2}{2^2}$$

Substitute into the trig identity:

$$\cos^2 t + \sin^2 t = 1$$
$$\frac{(x-4)^2}{3^2} + \frac{(y+5)^2}{2^2} = 1$$

And we can see this is an ellipse with center $(4, -5)$ and $a = 3$ and $b = 2$.

**3.** Find a parameterization of the line segment between the points $(1, 2)$ and $(-4, 5)$.

$$x = (1-t)1 + t(-4) = 1 - 5t$$
$$y = (1-t)2 + t(5) = 2 + 3t, \quad 0 \le t \le 1$$

**4.** Sketch the parametric curve

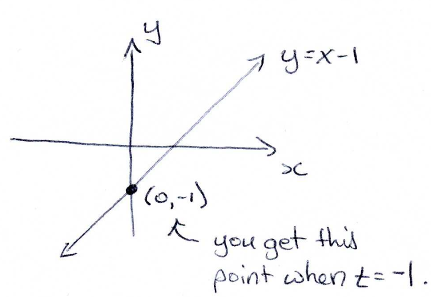
$$x = 1 + t$$
$$y = t$$

by eliminating the parameter.

$\left.\begin{aligned} x &= 1+t \\ y &= t \end{aligned}\right\}$ put second equation into first.

$x = 1 + y$

$y = x - 1$



**5.** Sketch the parametric curve

$$x = 5 - 3t$$
$$y = 2 + t$$
$$-1 \le t \le 3$$

by eliminating the parameter.

$x = 5 - 3t$
$y = 2 + t$ } Solve $2^{ND}$ equation for $t$, sub into $1^{st}$ equation.
$-1 \le t \le 3$

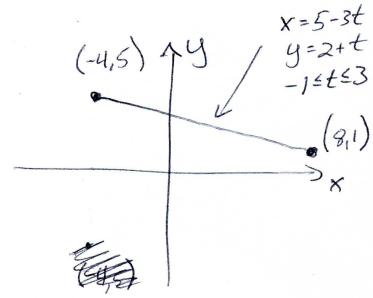
$x = 5 - 3(y - 2)$

$x = 5 - 3y + 6$

$x = 11 - 3y$

$y = -\frac{1}{3}x + \frac{11}{3}$

straight line.




Endpoints : $t = -1$ : $x = 5 - 3(-1) = 8$ $\quad (8,1)$
$y = 2 + (-1) = 1$

$t = 3$ : $x = 5 - 3(3) = -4$ $\quad (-4,5)$
$y = 2 + (3) = 5$

Both two these points are on $y = \frac{-1}{3}x + \frac{11}{3}$, so we probably simplified correctly.

**6.** Sketch the parametric curve

$$x = t - 3$$
$$y = 2/t$$
$$|t| \le 5$$

by eliminating the parameter.

$x = t - 3$
$y = 2/t$ } $2^{ND}$ Equation $t = 2/y$ sub into $1^{st}$ Equation
$|t| \le 5$

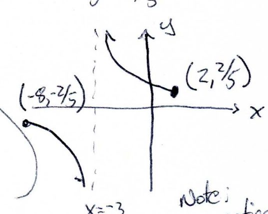
$x = \frac{2}{y} - 3$

$x + 3 = \frac{2}{y}$

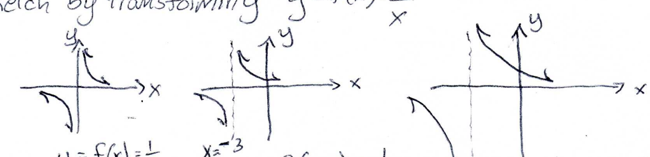
$y = \frac{2}{x+3}$

Endpoints: $t = -5$ : $x = (-5) - 3 = -8$
$y = 2/(-5) = -2/5$
$(-8, -2/5)$

$t = 5$ : $x = (5) - 3 = 2$
$y = 2/5$




Sketch by transforming $y = f(x) = \frac{1}{x}$




$y = f(x) = \frac{1}{x}$ $\qquad$ $y = f(x+3) = \frac{1}{x+3}$ (horizontal shift left) $\qquad$ $y = 2f(x+3) = \frac{2}{x-3}$ (vertical stretch)

Note: the vertical asymptote occurs at $t=0$, where $x=-3$.

**7.** Sketch the parametric curve

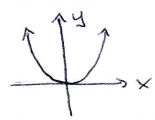
$$x = 2t^2 - 1$$
$$y = t^4$$

by eliminating the parameter.

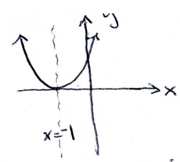
$x = 2t^2 - 1$
$y = t^4$
Solve $1^{st}$ equation for $t^2 = \frac{x+1}{2}$
sub into second:

$y = (t^2)^2$
$= \left(\frac{x+1}{2}\right)^2$
$= \frac{1}{4}(x+1)^2$

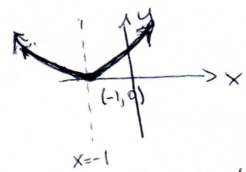
sketch by transforming




$y = f(x) = x^2$




$y = f(x+1) = (x+1)^2$
horizontal shift left one unit.




$y = \frac{1}{4}f(x+1) = \frac{1}{4}(x+1)^2$
vertical compression by a factor of 4.

**8.** For the parametric curve

$$x = at + b$$
$$y = ct + d$$

where $a$ and $c$ are not both zero.

(a) Eliminate the parameter $t$ and explain why its graph is a line.
(b) Find the slope, $y$-intercept, and $x$-intercept if they exist.
(c) Under what conditions would the line be horizontal? Vertical?

$x = at + b \quad a \neq 0$
$y = ct + d \quad c \neq 0.$

Eliminate $t$:

$t = \frac{x-b}{a} = \frac{y-d}{c}$

Notice this is where $a \neq 0$, and $c \neq 0$ must occur to avoid division by zero. Let's solve for $y$:

Note: one of $a, b$ could be zero, just not both.

$y = \frac{c}{a}(x-b) + d$
$= \frac{c}{a}x + d - \frac{cb}{a}.$

straight line (linear) with slope $\frac{c}{a}$ and $y$-intercept $d - \frac{cb}{a}$.

The $x$ intercept is when

$0 = \frac{c}{a}x + d - \frac{cb}{a}$

$x = \frac{a}{c}\left(\frac{cb}{a} - d\right) = b - \frac{da}{c}.$

The line is horizontal when $c = 0$ and $a \neq 0$. (This avoids the indeterminant form $\frac{0}{0}$).

The line is vertical when $a = 0$ and $c \neq 0$.

The fact that the parametric representation contains the vertical line makes this a powerful representation of a line.

**9.** For the parametric curve

$$x = tc + (1-t)a$$
$$y = td + (1-t)b$$
$$0 \leq t \leq 1$$

(a) Determine the value of $t$ that divides the line into two equal segments. (b) Determine the value of $t$ that divides the line into three equal segments. (c) What do you think the values of $t$ should be to split the line into $n$ equal segments?

$x = tc + (1-t)a$
$y = td + (1-t)b$
$0 \leq t \leq 1$

at $t=0$, $(x,y) = (a,b)$
$t=1$, $(x,y) = (c,d)$

at $t = \frac{1}{2}$, $(x,y) = \left(\frac{c+a}{2}, \frac{d+b}{2}\right)$

ie) midpoint of line.

at $t = \frac{1}{3}$, $(x,y) = \left(\frac{c+2a}{3}, \frac{d+2b}{3}\right)$

$t = \frac{2}{3}$, $(x,y) = \left(\frac{2c+a}{3}, \frac{2d+b}{3}\right)$

ie) $t = \frac{1}{3}, \frac{2}{3}$ split line into three equal pieces.

It looks like we can generalize. If we want to split the line into $n$ equal pieces, use

$t = \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, \frac{n-1}{n}$.