Questions

1. Assuming x, y, and z are positive, use properties of logarithms to write the expression as a single logarithm.

$3\ln(x^3y) + 2\ln(yz^2) - \ln(xyz)$

2. Sketch the function $f(x) = 5\ln(1-x) - 4$ by transforming the basic function $y = \ln x$. Then analyze it for domain, range, continuity, increasing or decreasing behaviour, symmetry, boundedness, extrema, asymptotes, and end behaviour.

Solutions

1. Assuming x, y, and z are positive, use properties of logarithms to write the expression as a single logarithm.

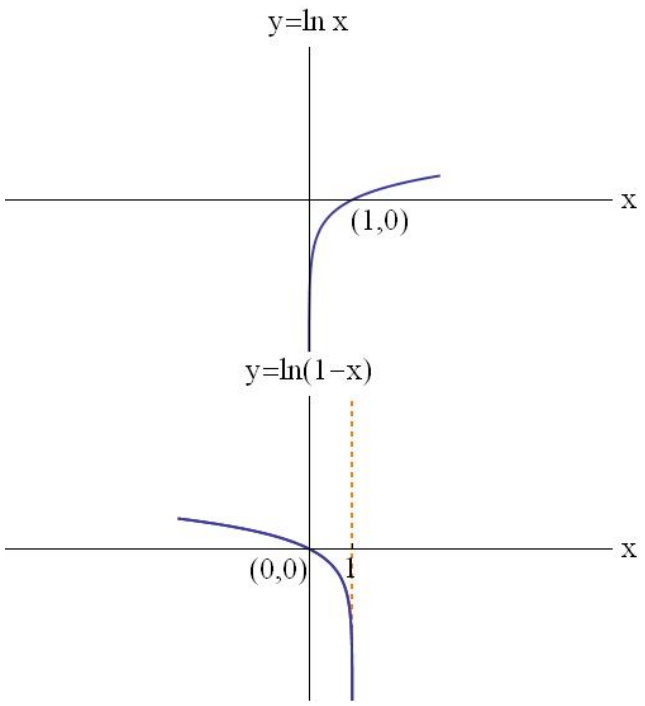
$3\ln(x^3y) + 2\ln(yz^2) - \ln(xyz)$

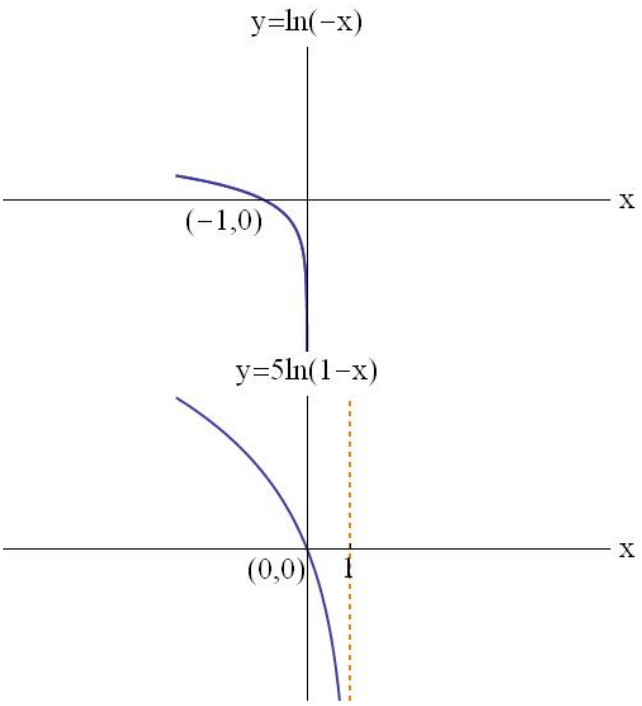
$$\begin{aligned}
3\ln(x^3y) + 2\ln(yz^2) - \ln(xyz) &= \ln((x^3y)^3) + \ln((yz^2)^2) - \ln(xyz) \\
&= \ln(x^9y^3) + \ln(y^2z^4) - \ln(xyz) \\
&= \ln((x^9y^3)(y^2z^4)) - \ln(xyz) \\
&= \ln(x^9y^5z^4) - \ln(xyz) \\
&= \ln\left(\frac{x^9y^5z^4}{xyz}\right) \\
&= \ln\left(x^8y^4z^3\right)
\end{aligned}$$

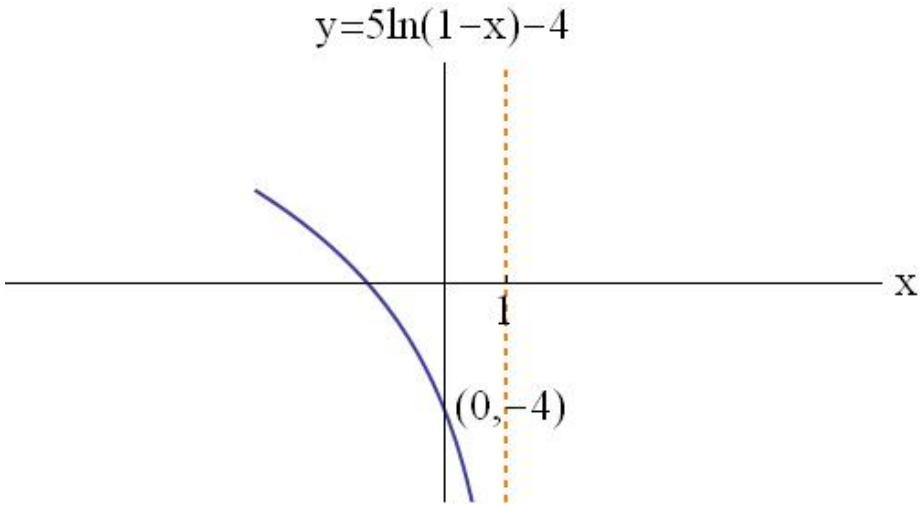
2. Sketch the function $f(x) = 5\ln(1-x) - 4$ by transforming the basic function $y = \ln x$. Then analyze it for domain, range, continuity, increasing or decreasing behaviour, symmetry, boundedness, extrema, asymptotes, and end behaviour.

Here are the algebraic representations of the transformations:
Basic function: $y = f(x) = \ln x$.
Flip about the y-axis: $y = f(-x) = \ln(-x)$.
Shift right 1 unit: $y = f(1-x) = \ln(1-x)$. (normally adding one would be a shift left, but since the x has a minus sign this is a shift right)
Stretch vertically by a factor of 5: $y = 5f(1-x) = 5\ln(1-x)$.
Shift down by a factor of 4: $y = 5f(1-x) - 4 = 5\ln(1-x) - 4$.

Here are the sketches:







Domain: $x \in (-\infty, 1)$
Range: $y \in (-\infty, \infty)$
Continuity: continuous for all x in the domain, $x \in (-\infty, 1)$
Increasing-decreasing behaviour: decreasing for all $x \in (-\infty, 1)$
Symmetry: none
Boundedness: not bounded
Local Extrema: none
Horizontal Asymptotes: none
Vertical Asymptotes: $x = 1$
End behaviour: $\lim_{x \to -\infty} (5\ln(1-x) - 4) = \infty$ and $\lim_{x \to \infty} (5\ln(1-x) - 4)$ is not defined.