Questions

1. Graph by hand the solution to the system of inequalities, and determine the points of intersection for the region.

$$\begin{aligned} x^2 + y^2 &\le 9 \\ -x + y^2 - 1 &\le 0 \end{aligned}$$

2. Graph by hand the solution to the system of inequalities:

$$\begin{aligned} 5x - 3y &> 1 \\ 3x + 4y &\le 18 \end{aligned}$$

3. Graph by hand the solution to the system of inequalities, and determine the points of intersection for the region.

$$\begin{aligned} 2x + y &\le 80 \\ x + 2y &\le 80 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

4. Graph by hand the solution to the system of inequalities, and determine the points of intersection for the region. As best you can, describe the region in terms of the values y takes as x varies across the region (this will be an important skill in Calculus II).

$$\begin{aligned} x^2 + y^2 &\le 4 \\ y &\ge |x| \end{aligned}$$

5. Graph by hand the solution to the system of inequalities, and determine the points of intersection for the region. As best you can, describe the region in terms of the values y takes as x varies across the region.

$$\begin{aligned} y &\ge 4x - x^2 \\ y &\le 8x - 2x^2 \end{aligned}$$

6. A developer has 100 acres that is being divided into 1 acre lots, each with a new construction home. He is going to have two basic types of houses on the lots (with small variations in each type), a modest or a deluxe house. The building costs are \$20,000 on average for the modest house, \$40,000 for the deluxe. The profits are projected to be \$35,000 on average for the modest house, \$50,000 for the deluxe. The developer has \$2.6 million available for the project. How many of each type of house should the developer build to maximize profits?

Solutions

1. Graph by hand the solution to the system of inequalities, and determine the points of intersection for the region.

$$x^2 + y^2 < 9$$
$$-x + y^2 - 1 \leq 0$$

Sketch $-x+y^2-1=0$:

$y^2 = x+1$ is a quadratic that opens to the right, shifted left 1 unit.

Test Point: $(0,0)$
$-x+y^2-1 \leq 0$
$-(0)+(0)^2-1 \leq 0$
$-1 \leq 0$ True!
shade side of line with $(0,0)$.

Sketch $x^2+y^2=9$:

$x^2+y^2=3^2$
circle of radius 3 centered at the origin.

Test Point $(0,0)$
$(0)^2+(0)^2 \leq 9$
$0 \leq 9$ True!
shade inside of circle

Points of intersection:

Solve $\left.\begin{array}{l} y^2=x+1 \\ x^2+y^2=9 \end{array}\right\}$ system of equations.

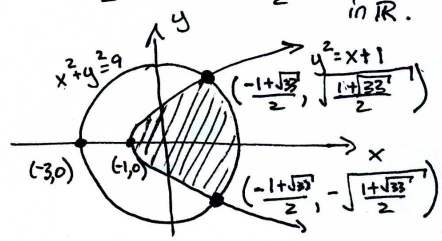
$\Rightarrow x^2+x+1=9$
$x^2+x-8=0$
$x = \dfrac{-1 \pm \sqrt{1-4(1)(-8)}}{2(1)} = \dfrac{-1 \pm \sqrt{33}}{2}$

Use $y^2 = x+1$ to find (x,y) pairs:

If $x = \dfrac{-1+\sqrt{33}}{2}$, $y^2 = \dfrac{-1+\sqrt{33}}{2}+1$
$y^2 = \dfrac{1+\sqrt{33}}{2}$
$y = \pm\sqrt{\dfrac{1+\sqrt{33}}{2}}$

Two points of intersection:
$\left(\dfrac{-1+\sqrt{33}}{2}, +\sqrt{\dfrac{1+\sqrt{33}}{2}}\right)$
$\left(\dfrac{-1+\sqrt{33}}{2}, -\sqrt{\dfrac{1+\sqrt{33}}{2}}\right)$

If $x = \dfrac{-1-\sqrt{33}}{2}$, $y^2 = \dfrac{1-\sqrt{33}}{2} < 0$ no y values in $\mathbb{R}$.



2. Graph by hand the solution to the system of inequalities:

$$5x - 3y > 1$$
$$3x + 4y \leq 18$$

Sketch $5x-3y=1$:

If $x=0$, then $-3y=1$
$y=-1/3$
$\Rightarrow (0,-1/3)$.

If $y=0$, then $5x=1$
$x=1/5$
$\Rightarrow (1/5,0)$.

Dashed line since we want strictly greater than: $5x-3y>1$.

Test Point is $(0,0)$:
$5(0)-3(0)>1$
$0>1$ False!

shade side of line that does not contain $(0,0)$.

Sketch $3x+4y \leq 18$:

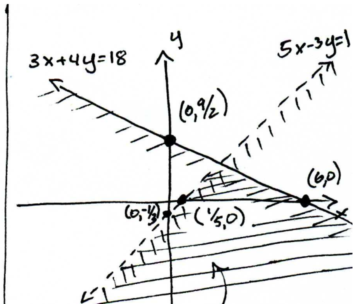
If $x=0$, then $4y=18$
$y=9/2$
$\Rightarrow (0,9/2)$

If $y=0$, then $3x=18$
$x=6$
$\Rightarrow (6,0)$.

Solid line since we want less than or equal, $3x+4y \leq 18$.

Test Point is $(0,0)$:
$3(0)+4(0) \leq 18$
$0 \leq 18$ True!

shade side of line that contains $(0,0)$.



this region is solution to
$5x-3y>1$
$3x+4y \leq 18$

3. Graph by hand the solution to the system of inequalities, and determine the points of intersection for the region.

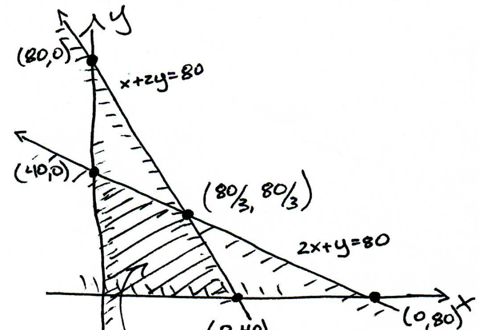
$$
\begin{aligned}
2x + y &\le 80\\
x + 2y &\le 80\\
x &\ge 0\\
y &\ge 0
\end{aligned}
$$

Sketch $2x+y=80$:

If $x=0$, then $y=80$.
$\Rightarrow (0,80)$.
If $y=0$, then $2x=80$
$x=40$
$\Rightarrow (40,0)$.
Solid line since
$2x+y \le 80$.
Test Point is $(0,0)$:
$2(0)+(0)\le 80$
$0\le 80$ True!
shade side of line containing $(0,0)$.

sketch $x+2y=80$:

If $x=0$, then $2y=80$
$y=40$
$\Rightarrow (0,40)$.
If $y=0$, then $x=80$
$\Rightarrow (80,0)$.
Solid line since
$x+2y\le 80$.
Test Point is $(0,0)$:
$(0)+2(0)\le 80$
$0\le 80$ True!
shade side of line containing $(0,0)$.



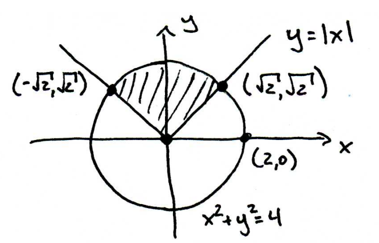
intersection point: $2x+4y=160$ (multiply equation by 2)
$\underline{2x+y\ \ =80}$ (subtract other equation)
$3y=80$
$\rightarrow y=\frac{80}{3}$. Then $x=80-2y=80-2\left(\frac{80}{3}\right)=\frac{80}{3}$
$\Rightarrow$ intersection is at $(80/3, 80/3)$

4. Graph by hand the solution to the system of inequalities, and determine the points of intersection for the region. As best you can, describe the region in terms of the values y takes as x varies across the region (this will be an important skill in Calculus II).

$$
\begin{aligned}
x^2 + y^2 &\le 4\\
y &\ge |x|
\end{aligned}
$$

Sketch $x^2+y^2=4$:
Circle radius 2 centered at origin.
Test Point $(0,0)$:
$(0)^2+(0)^2\le 4$
$0\le 4$ True!
shade inside of circle

sketch $y=|x|$
(one of basic functions)
Test Point $(0,1)$:
$1 \ge |0|$ True!
shade side of line containing $(0,1)$



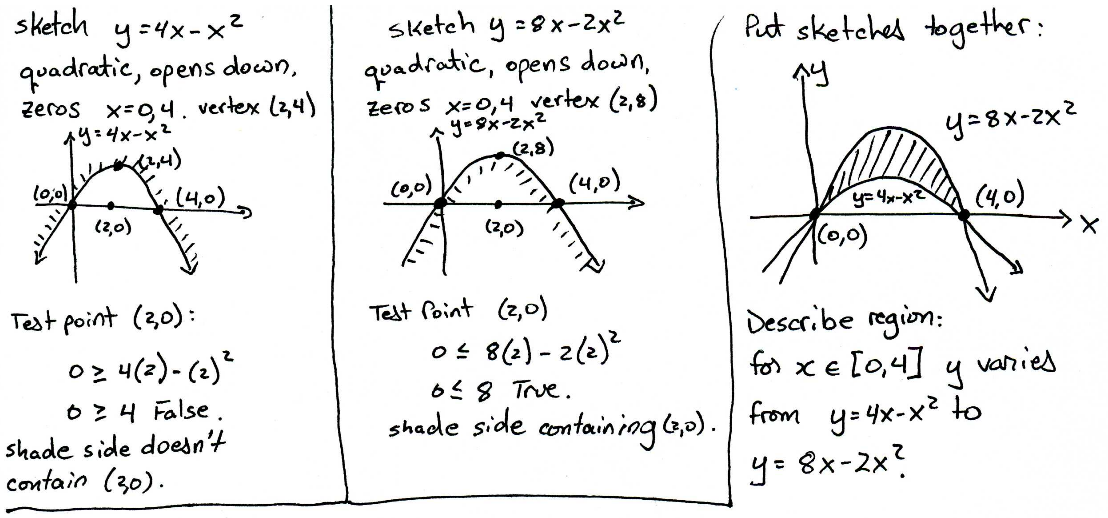
Intersection: $\left.\begin{aligned} x^2+y^2&=4\\ y&=|x|\end{aligned}\right\}$ system of equations.

$x^2+(|x|)^2=4$
$2x^2=4$
$x^2=2$
$x=\pm\sqrt{2}$
If $x=+\sqrt{2}$: $y=|\sqrt{2}|=\sqrt{2}\Rightarrow(\sqrt{2},\sqrt{2})$.
$x=-\sqrt{2}$: $y=|-\sqrt{2}|=\sqrt{2}\Rightarrow(-\sqrt{2},\sqrt{2})$.

Description of region:
for $x\in[-\sqrt{2},\sqrt{2}]$,
y varies from
$y=|x|$ to $y=\sqrt{4-x^2}$.

5. Graph by hand the solution to the system of inequalities, and determine the points of intersection for the region. As best you can, describe the region in terms of the values y takes as x varies across the region.

$y \geq 4x - x^2$
$y \leq 8x - 2x^2$



6. A developer has 100 acres that is being divided into 1 acre lots, each with a new construction home. He is going to have two basic types of houses on the lots (with small variations in each type), a modest or a deluxe house. The building costs are \$20,000 on average for the modest house, \$40,000 for the deluxe. The profits are projected to be \$35,000 on average for the modest house, \$50,000 for the deluxe. The developer has \$2.6 million available for the project. How many of each type of house should the developer build to maximize profits?

Here is a mixture chart for this problem. A mixture chart is an intermediate step from the words of the problem to the mathematical equations, and can be left out if you can determine the mathematical equations without it–if you do leave it out, be sure to define your variables in your solution!

	Space (100 acres available)	Build Cost (\$2,600,000)	Minimum that must be made	Profit
Modest House, x	1 acre	\$20,000	0	\$35,000
Deluxe House, y	1 acre	\$40,000	0	\$50,000

Now we need to translate <u>all</u> of the information in the mixture chart into some mathematical formulas.

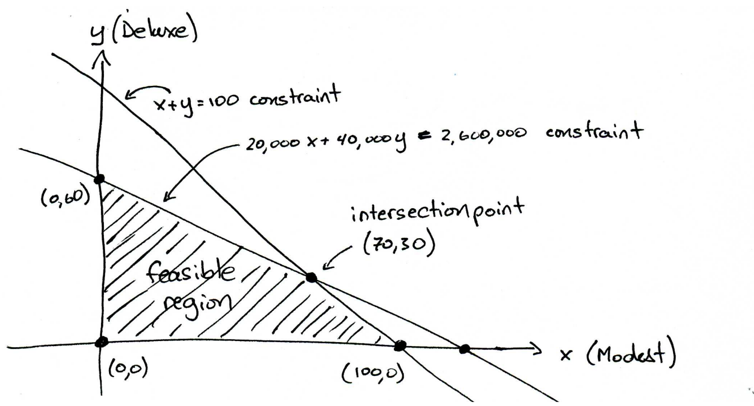
Constraints:

$$x > 0, y > 0 \qquad \text{(can't build a "negative" number of houses!)}$$
$$x + y \leq 100 \text{ (space)}$$
$$20,000x + 40,000y \leq 2,600,000 \text{ (available capital)}$$

Profit:

$$P = 35,000x + 50,000y$$

Let's sketch the feasible region for this problem!



The maximum profit will occur at one of the corner points of the feasible region, so we need to work all those out. We have them all except where the two constraints intersect.

$$x + y = 100 \quad (1)$$
$$20,000x + 40,000y = 2,600,000 \quad (2)$$

From Eq. (1), we have $x = 100 - y$. Substitute this into Eq. (2) to get

$$\begin{aligned} 20,000x + 40,000y &= 2,600,000 \\ 20,000(100 - y) + 40,000y &= 2,600,000 \\ 2,000,000 - 20,000y + 40,000y &= 2,600,000 \\ 20,000y &= 600,000 \\ 2y &= 60 \\ y &= 30 \end{aligned}$$

And therefore, $x = 100 - y = 100 - 30 = 70$. So the corner point is $(70, 30)$.

So all we need to do is calculate the profit at all the corner points,

Corner Point	Profit: $P = 35,000x + 50,000y$
$x = 0, y = 0$	$P = 35,000(0) + 50,000(0) = \$0$
$x = 0, y = 65$	$P = 35,000(0) + 50,000(65) = \$3,250,000$
$x = 100, y = 0$	$P = 35,000(100) + 50,000(0) = \$3,500,000$
$x = 70, y = 30$	$P = 35,000(70) + 50,000(30) = \$3,950,000$

So the developer should make 70 modest homes, and 30 deluxe homes, and will receive a profit of \$3.95 million dollars.

If you were to plot the maximum profit equation, $P = 35,000x + 50,000y = 3,950,000$ you would see that it intersects the feasible region only at the point $(70, 30)$, as expected.