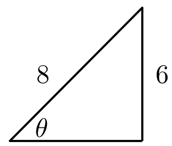
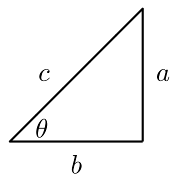
Questions

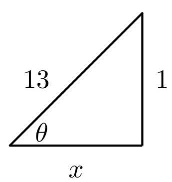
1. Find the value of all six of the trigonometric functions of the angle θ given the following right angle triangle. Note the triangle is not drawn to scale.



2. Using the labeling of the triangle below, prove that if θ is an acute angle in any right triangle, then $(\sin\theta)^2+(\cos\theta)^2 = 1$.



3. Find the value of all six of the trigonometric functions of the angle θ given the following right angle triangle. Note the triangle is not drawn to scale.



4. Assume that θ is an acute angle in a right triangle which satisfies $\sec\theta = \dfrac{7}{6}$. Evaluate the remaining trigonometric functions of the angle θ.

5. What are the values of the following? I would recommend you know how to work them out from the special triangles, but if you want to you can memorize them.

(a) $\sin(\pi/3) =$

(b) $\tan(\pi/3) =$

(c) $\csc(\pi/4) =$

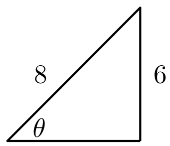
(d) $\sec(\pi/4) =$

(e) $\cos(\pi/6) =$

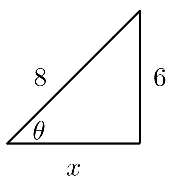
(f) $\cot(\pi/6) =$

Solutions

1. Find the value of all six of the trigonometric functions of the angle θ given the following right angle triangle. Note the triangle is not drawn to scale.



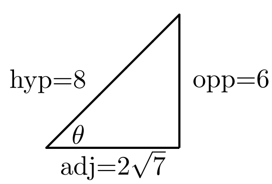
First, we need to use the Pythagorean theorem to find the length of the side adjacent to θ in the triangle, which I have labelled x:



$$x^2 + 6^2 = 8^2 \qquad \Rightarrow \qquad x^2 = 64 - 36 = 28 \qquad \Rightarrow \qquad x = \sqrt{28} = 2\sqrt{7}$$

We choose the positive root, not the negative root, since the angle is acute.

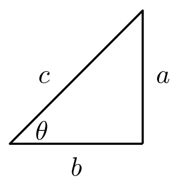
The triangle can be labeled as



From the reference triangle, we get value of the six trig functions:

$$\begin{aligned}
\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{8} = \frac{3}{4} \\
\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{7}}{8} = \frac{\sqrt{7}}{4} \\
\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{2\sqrt{7}} = \frac{3}{\sqrt{7}} \\
\csc\theta &= \frac{\text{hypotenuse}}{\text{opposite}} = \frac{8}{6} = \frac{4}{3} \\
\sec\theta &= \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{8}{2\sqrt{7}} = \frac{4}{\sqrt{7}} \\
\cot\theta &= \frac{\text{adjacent}}{\text{opposite}} = \frac{2\sqrt{7}}{6} = \frac{\sqrt{7}}{3}
\end{aligned}$$

2. Using the labeling of the triangle below, prove that if θ is an acute angle in any right triangle, then $(\sin\theta)^2+(\cos\theta)^2 = 1$.



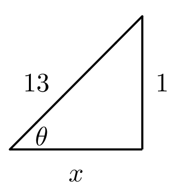
We need to work out sine and cosine of the angle θ:

$$\begin{aligned}\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}\\ \cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}\end{aligned}$$

Now we need to show the following quantity reduces to 1 (break out the algebra!):

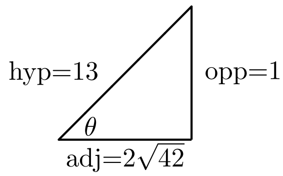
$$\begin{aligned}\sin^2\theta + \cos^2\theta &= (\sin\theta)^2 + (\cos\theta)^2\\ &= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2\\ &= \frac{a^2}{c^2} + \frac{b^2}{c^2}\\ &= \frac{a^2+b^2}{c^2}\\ &= \frac{c^2}{c^2} \text{ (by Pythagorean theorem, } a^2+b^2=c^2\text{)}\\ &= 1\end{aligned}$$

3. Find the value of all six of the trigonometric functions of the angle θ given the following right angle triangle. Note the triangle is not drawn to scale.



First, we need to use the Pythagorean theorem to find the length of the side adjacent to θ in the triangle, which I have labelled x:

$$x^2 + 1^2 = 13^2 \qquad \Rightarrow \qquad x^2 = 169 - 1 = 168 \qquad \Rightarrow \qquad x = \sqrt{168} = 2\sqrt{42}$$

The triangle can be labeled as 

From the reference triangle, we get value of the six trig functions:

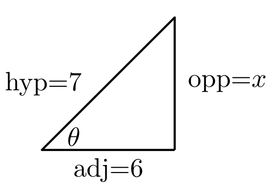
$$\begin{aligned}
\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{13} \\
\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{42}}{13} \\
\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{2\sqrt{42}} \\
\csc\theta &= \frac{\text{hypotenuse}}{\text{opposite}} = \frac{13}{1} = 13 \\
\sec\theta &= \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{13}{2\sqrt{42}} \\
\cot\theta &= \frac{\text{adjacent}}{\text{opposite}} = \frac{2\sqrt{42}}{1} = 2\sqrt{42}
\end{aligned}$$

4. Assume that θ is an acute angle in a right triangle which satisfies $\sec\theta = \frac{7}{6}$. Evaluate the remaining trigonometric functions of the angle θ.

To begin, we need to draw the right triangle with θ in it, and then use that to help us evaluate the trigonometric functions.

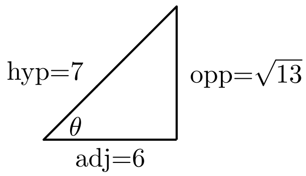
The triangle is based on the relation we have been given, and use SOH-CAH-TOA to get determine how to label:

$$\sec\theta = \frac{7}{6} = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{adjacent}}.$$



We use the Pythagorean theorem to determine the length of the opposite side.

$$7^2 = 6^2 + x^2 \qquad \Rightarrow \qquad x^2 = 49 - 36 = 13 \qquad \Rightarrow \qquad x = \sqrt{13}$$



Our triangle becomes:

From the reference triangle, we get value of the six trig functions:

$$
\begin{aligned}
\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{13}}{7} \\
\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{6}{7} \\
\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} = \frac{\sqrt{13}}{6} \\
\csc\theta &= \frac{\text{hypotenuse}}{\text{opposite}} = \frac{7}{\sqrt{13}} \\
\sec\theta &= \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{7}{6} \\
\cot\theta &= \frac{\text{adjacent}}{\text{opposite}} = \frac{6}{\sqrt{13}}
\end{aligned}
$$

5. What are the values of the following? I would recommend you know how to work them out from the special triangles, but if you want to you can memorize them.

(a) $\sin(\pi/3) = \dfrac{\sqrt{3}}{2}$

(b) $\tan(\pi/3) = \sqrt{3}$

(c) $\csc(\pi/4) = \sqrt{2}$

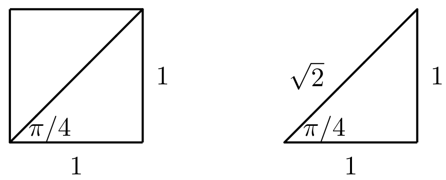
(d) $\sec(\pi/4) = \sqrt{2}$

(e) $\cos(\pi/6) = \dfrac{\sqrt{3}}{2}$

(f) $\cot(\pi/6) = \sqrt{3}$

Start by writing down the two special triangles, and then use SOH-CAH-TOA and the reciprocal definitions to read off the values. The details are on the following page.

A 45-45-90 Triangle

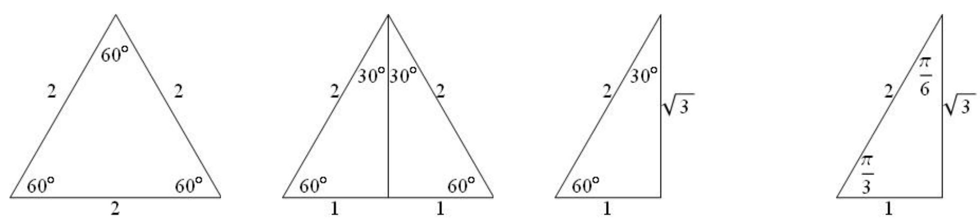


$$\sin\left(\frac{\pi}{4}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} \qquad \csc\left(\frac{\pi}{4}\right) = \frac{1}{\sin\left(\frac{\pi}{4}\right)} = \sqrt{2}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}} \qquad \sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \sqrt{2}$$

$$\tan\left(\frac{\pi}{4}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{1} = 1 \qquad \cot\left(\frac{\pi}{4}\right) = \frac{1}{\tan\left(\frac{\pi}{4}\right)} = 1$$

A 30-60-90 Triangle



$$\sin\left(\frac{\pi}{3}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2} \qquad \csc\left(\frac{\pi}{3}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = \frac{2}{\sqrt{3}}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{\text{adj}}{\text{hyp}} = \frac{1}{2} \qquad \sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = 2$$

$$\tan\left(\frac{\pi}{3}\right) = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{1} = \sqrt{3} \qquad \cot\left(\frac{\pi}{3}\right) = \frac{1}{\tan\left(\frac{\pi}{3}\right)} = \frac{1}{\sqrt{3}}$$

$$\sin\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2} \qquad \csc\left(\frac{\pi}{6}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = 2$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2} \qquad \sec\left(\frac{\pi}{6}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{2}{\sqrt{3}}$$

$$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} \qquad \cot\left(\frac{\pi}{6}\right) = \frac{1}{\tan\left(\frac{\pi}{3}\right)} = \frac{\sqrt{3}}{1} = \sqrt{3}$$