Part I: Short Answer

1. Solve $ax + b = c$ for x.

$$\begin{aligned}
ax + b &= c \quad \text{(begin by writing the equation you are given)} \\
ax + b - b &= c - b \quad \text{(subtract } b \text{ from both sides to isolate } ax\text{)} \\
ax &= c - b \quad \text{(simplify)} \\
\frac{1}{a}(ax) &= \frac{1}{a}(c - b) \quad \text{(to isolate the } x\text{, multiply both sides of the equation by } \tfrac{1}{a}\text{)} \\
x &= \frac{c - b}{a} \quad \text{(simplify)}
\end{aligned}$$

There is more information on solving equations that involve variables on the purplemath page.

2. Solve $\dfrac{1}{ax + b} = c$ for x.

$$\begin{aligned}
\frac{1}{ax + b} &= c \quad \text{(begin by writing the equation you are given)} \\
\left(\frac{1}{ax + b}\right)(ax + b) &= c(ax + b) \quad \text{(multiply both sides by } ax + b \text{ to make the denominator 1)} \\
1 &= cax + bc \quad \text{(simplify)} \\
1 - bc &= cax + bc - bc \quad \text{(subtract } bc \text{ from both sides to isolate } cax\text{)} \\
1 - bc &= cax \quad \text{(simplify)} \\
\frac{1}{ca}(1 - bc) &= \frac{1}{ca}(cax) \quad \text{(to isolate the } x\text{, multiply both sides of the equation by } \tfrac{1}{ca}\text{)} \\
x &= \frac{1 - bc}{ca} \quad \text{(simplify)}
\end{aligned}$$

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3. Express the following with a common denominator and simplify as much as possible: $\dfrac{4}{x + 1} - \dfrac{5}{x - 1}$.

$$\begin{aligned}
\frac{4}{x + 1} - \frac{5}{x - 1} &= \left(\frac{4}{x + 1}\right)(1) - \left(\frac{5}{x - 1}\right)(1) \quad \text{(multiply the two terms by 1)} \\
&= \left(\frac{4}{x + 1}\right)\left(\frac{x - 1}{x - 1}\right) - \left(\frac{5}{x - 1}\right)\left(\frac{x + 1}{x + 1}\right) \\
&\quad \text{(replace the 1 by quantities that will lead to a common denominator)} \\
&= \frac{4(x - 1) - 5(x + 1)}{(x + 1)(x - 1)} \quad \text{(rewrite with the common denominator)} \\
&= \frac{4x - 4 - 5x - 5}{(x + 1)(x - 1)} \quad \text{(multiply the polynomials in the numerator out)} \\
&= \frac{-x - 9}{(x + 1)(x - 1)} \quad \text{(simplify)}
\end{aligned}$$

There is more information on polynomial multiplication on the purplemath page.

4. Expand $(3x-4)^2$.

$$(3x-4)^2 = (3x)^2 + (4)^2 - 2(3x)(4) = 9x^2 + 16 - 24x$$

There is more information on polynomial multiplication on the purplemath page.

5. Solve $(x+13)^2 = 16$ for x.

$$\begin{aligned}
(x+13)^2 &= 16 \quad \text{(begin by writing the equation you are given)} \\
\sqrt{(x+13)^2} &= \sqrt{16} \quad \text{(take the square root of both sides to isolate } x+13\text{)} \\
|x+13| &= 4 \quad \text{(this is using the definition of the absolute value function)} \\
x+13 &= \pm 4 \quad \text{(remove the absolute values, introducing the } \pm\text{)} \\
x+13-13 &= -13 \pm 4 \quad \text{(subtract 13 from both sides)} \\
x &= -13 \pm 4 \quad \text{(simplify)} \\
x = -13+4 &\quad \text{or} \quad x = -13-4 \quad \text{(write as two solutions)} \\
x = -9 &\quad \text{or} \quad x = -17 \quad \text{(simplify)}
\end{aligned}$$

There is more information on working with quadratics on the purplemath page.

6. Solve $-2-2a = -2-2(x+y)$ for x.

$$\begin{aligned}
-2-2a &= -2-2(x+y) \quad \text{(begin by writing the equation you are given)} \\
+2-2-2a &= +2-2-2(x+y) \quad \text{(add 2 to both sides)} \\
-2a &= -2(x+y) \quad \text{(simplify)} \\
\frac{-2a}{-2} &= \frac{-2(x+y)}{-2} \quad \text{(divide both sides by } -2 \text{ to isolate } x+y\text{)} \\
a &= x+y \quad \text{(simplify)} \\
a-y &= x+y-y \quad \text{(subtract } y \text{ from both sides)} \\
a-y &= x \quad \text{(simplify)}
\end{aligned}$$

There is more information on solving equations that involve variables on the purplemath page.

Part II: True or False

1. $\dfrac{1}{a+b} = \dfrac{1}{a} + \dfrac{1}{b}$ .... T $\boxed{\text{F}}$

The answer here is False. If you answered true, your error is in violating the order of operations. The original expression cannot be simplified; think of the denominator as having brackets around it, and the expression reads in English as "Add a and b, then take the reciprocal".

$$\frac{1}{a+b} = \frac{1}{(a+b)}$$

2. $-a(4+x) = -4a + xa$ .... T $\boxed{\text{F}}$

You have to distribute the $-a$ into each term in the $4+x$ (mathematically, we say *multiplication distributes over addition*), so we get

$$-a(4+x) \quad = \quad (-a)(4) + (-a)(x) = -4a - ax$$

There is more information on the distributive property on the purplemath page.

3. $\sqrt{x^2+16} = x+4$ .... T $\boxed{\text{F}}$

This is False, and, unfortunately, an extremely common error that students make.

In the Problems 11 and 12, we used the fact that multiplication distributes over addition. If you answered True to this problem, you assumed that taking the square root distributes over addition, which is not true. Taking the square root is not an arithmetical operation (addition, subtraction, multiplication, division are what I refer to as arithmetical operations), it is a functional operation, and functional operations in general do not distribute over arithmetical operations.

To help you remember this fact, always think of

$$\sqrt{a+b} = \sqrt{(a+b)}$$

which reminds you that the addition must be done first, and so the expression cannot be simplified.

4. $\dfrac{(x+1)(3x+27)+x^3}{x+1} = 3x+27+x^3$ .... T $\boxed{\text{F}}$

This is False, since the expression $x+1$ has not been factored out of the term x^3. Here is what you can write:

$$\begin{aligned}
\frac{(x+1)(3x+27)+x^3}{x+1} &= \frac{(x+1)(3x+27)+\frac{(x+1)}{(x+1)}x^3}{x+1} \quad \text{(insert } 1 = (x+1)/(x+1) \text{ in smart manner)} \\
&= \frac{{\color{blue}(x+1)}\left((3x+27)+\frac{1}{(x+1)}x^3\right)}{{\color{blue}x+1}} \quad \text{(factor } (x+1) \text{ out of each term in the numerato} \\
&= (3x+27)+\frac{1}{(x+1)}x^3 \quad \text{(cancel terms in blue)} \\
&= 3x+27+\frac{x^3}{(x+1)} \quad \text{(simplify)}
\end{aligned}$$

There is an additional wrinkle to this, which is that when we wrote $\frac{x+1}{x+1} = 1$, we excluded from our expression the point $x+1=0$, or $x=-1$. We will talk more about this wrinkle in class.

5. $2\sqrt{x+y} = \sqrt{4x+4y}$ . . . . $\boxed{\text{T}}$ F

$$\begin{aligned} 2\sqrt{x+y} &= \sqrt{4}\sqrt{x+y} \\ &= \sqrt{(4)(x+y)} \\ &= \sqrt{4x+4y} \end{aligned}$$

There is more information on how to work correctly with radicals on the purplemath page.

Notice that in this case we were able to distribute the square root (a functional operator) over the multiplication. This is allowed only because the quantities were both positive! Here is an example with negative quantities that leads to a contradiction (this requires you know a little bit about complex numbers, so if you don't quite follow this that's ok):

$$1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1}\cdot\sqrt{-1} = i \cdot i = i^2 = -1$$

Stop the world! We have just shown $1 = -1$–actually, we haven't. We have made an error when we wrote $\sqrt{(-1)(-1)} = \sqrt{-1}\cdot\sqrt{-1}$.

Don't worry about this too much–I will point out things like this where appropriate in the course, but they won't trouble us too much.

6. $\dfrac{\left(\dfrac{1}{a}\right)}{\left(\dfrac{1}{b}\right)} = \dfrac{b}{a}$ . . . . $\boxed{\text{T}}$ F

$$\begin{aligned} \frac{\left(\dfrac{1}{a}\right)}{\left(\dfrac{1}{b}\right)} &= \left(\frac{1}{a}\right)\left(\frac{1}{\left(\frac{1}{b}\right)}\right) \quad \text{(factor out the numerator)} \\ &= \left(\frac{1}{a}\right)(b) \quad \text{(instead of dividing, multiply by the reciprocal)} \\ &= \frac{b}{a} \quad \text{(simplify)} \end{aligned}$$

The basic result we have used here is that instead of dividing by something, we can multiply by the reciprocal of that something:

$$\begin{aligned} \frac{1}{\left(\frac{c}{d}\right)} &= \frac{1}{\left(\frac{c}{d}\right)}\cdot\frac{d}{d} \\ &= \frac{d}{\left(\frac{cd}{d}\right)} \\ &= \frac{d}{c} \end{aligned}$$

7. $\dfrac{\left(\dfrac{1}{a}\right)}{(b)} = \dfrac{1}{ab}$ .... $\boxed{\text{T}}$ F

$$\frac{\left(\dfrac{1}{a}\right)}{(b)} = \frac{\left(\dfrac{1}{a}\right)}{(b)} \cdot \frac{\left(\dfrac{1}{b}\right)}{\left(\dfrac{1}{b}\right)} = \frac{\left(\dfrac{1}{a}\right)\left(\dfrac{1}{b}\right)}{(b)\left(\dfrac{1}{b}\right)} = \frac{\left(\dfrac{1}{ab}\right)}{1} = \frac{1}{ab}$$

8. $x^2 - y^2 = (x-y)(x+y)$ .... $\boxed{\text{T}}$ F

This is the difference of squares.

There is more information on factoring techniques on the purplemath page.