Common Mathematical Errors in Precalculus

Many of these errors are algebraic or notational, yet these errors can prevent you from solving problems correctly.

Many of you will never make a single one of these errors, and most of you may only occasionally make one of these errors. Occasionally making one of these errors is not such a big deal. However, sometimes a student consistently makes these errors, and that *is* a big deal.

These are all serious errors, and you can expect to be penalized if you make them on tests.

Improper use of Functional Notation

Given $f(x) = x^2 - \sqrt{x^2 - 1}$, what is $f(-x)$, $f(3x-1)$, and $f(x+h)$?

The best way to get f correctly evaluated at $-x$, $3x-1$, and $x+h$ is to take your time when doing it! It means writing one extra step and not trying to do everything in your head.

Wherever there was an x, put blank parentheses, and then fill in the parentheses.

$$f(x) = x^2 - \sqrt{x^2 - 1}$$
$$f(\quad) = (\quad)^2 - \sqrt{(\quad)^2 - 1}$$
$$f(3x+1) = (3x+1)^2 - \sqrt{(3x-1)^2 - 1}$$
$$f(-x) = (-x)^2 - \sqrt{(-x)^2 - 1}$$
$$f(x+h) = (x+h)^2 - \sqrt{(x+h)^2 - 1}$$

Thinking Everything Commutes

Mathematicians say two operations *commute* if we can perform them in either order and get the same result. Commutivity means we can write things like $6 + 5 = 5 + 6$ and $2 \times 4 = 4 \times 2$. So, the integers commute under addition and multiplication. However, everything does not commute. Here are some common examples of algebraic errors that result from assuming that everything commutes. For each example, try to identify the two operations have been mistakenly assumed to commute (in the first example, it is the operation of addition and taking a square root–adding and then taking a square root is not the same as taking a square root and then adding).

Incorrect:	$\sqrt{x+y} = \sqrt{x} + \sqrt{y}$	Correct:	$\sqrt{x+y}$ can not be simplified
Incorrect:	$(x+y)^2 = x^2 + y^2$	Correct:	$(x+y)^2 = x^2 + y^2 + 2xy$
Incorrect:	$\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$	Correct:	$\frac{1}{x+y}$ can not be simplified
Incorrect:	$\sin(x+y) = \sin x + \sin y$	Correct:	$\sin(x+y) = \sin x \cos y + \cos x \sin y$
Incorrect:	$\sin 2x = 2 \sin x$	Correct:	$\sin 2x = 2 \sin x \cos x$
Incorrect:	$f(x+y) = f(x) + f(y)$	Correct:	$f(x+y)$ can not be simplified

Improper Algebraic Cancellation

Cancelling a term common to a numerator and denominator can only be done when the term can be factored out of the numerator and denominator. You can only cancel factors, not terms.

Improper cancellation can transform a problem that is solvable into one which is insolvable!

Incorrect: $\dfrac{(x+1)(3x+27)+x^3}{x+1} = \dfrac{\cancel{(x+1)}(3x+27)+x^3}{\cancel{x+1}} = 3x+27+x^3$

Incorrect: $\dfrac{h+x^3}{h} = \dfrac{\cancel{h}+x^3}{\cancel{h}} = x^3$

Correct: $\dfrac{(x+1)(3x+27)+x^3}{x+1} = \dfrac{\cancel{(x+1)}\left((3x+27)+\frac{x^3}{x+1}\right)}{\cancel{x+1}} = 3x+27+\dfrac{x^3}{x+1}$

Correct: $\dfrac{h+x^3}{h} = \dfrac{\cancel{h}\left(1+\frac{x^3}{h}\right)}{\cancel{h}} = 1+\dfrac{x^3}{h}$

Improper Use of = Sign

The equal sign should be read as "is equivalent to" in all uses. Some students have developed the habit of thinking the equal sign means "and next I do" or "this implies that", which is incorrect.

Here is an incorrect use of the equal sign:

Incorrect: $\dfrac{3x}{2} = 2 = 3x = 4 = x = \dfrac{4}{3}$

The problem here is that the equals sign is sometimes being used to mean "which implies" and sometimes to mean "is equivalent to". What the student who wrote the above is probably trying to say is this:

$\dfrac{3x}{2} = 2 \quad \underset{\text{which implies that}}{\Rightarrow} \quad 3x = 4 \quad \underset{\text{which implies that}}{\Rightarrow} \quad x = \dfrac{4}{3}$

This could be even better presented if it is written in the following manner, with notes to the side explaining what has been done.

$$\begin{aligned} \frac{3x}{2} &= 2 \\ 3x &= 4 \quad \text{multiply the equation by 2.} \\ x &= \frac{4}{3} \quad \text{solve for } x. \end{aligned}$$

The statements are what the student is doing in their head. Including short phrases that guide the reader (which is usually the person who wrote the solution!) through the process of obtaining the solution is a good idea sometimes. Many of the more interesting problems encountered cannot be solved in one line, and the solution may best be presented as groupings of shorter problems which can be combined to give the whole solution.

Improper Use of Brackets

Forgetting brackets can seriously alter a problem. Here is an example of incorrect mathematical statements, where the bracket is missing:

Incorrect: $-\left(\sqrt{-x+1}\right)^2 = - - x + 1 = x + 1$

The improper use of brackets frequently shows up, as it does in the problem above, as two algebraic operators written consecutively (i.e., $--$, $-+$, $\times+$, etc.). Anytime you check your solution and see two operators written consecutively, you have made an error! The proper solution retains the brackets as follows:

Correct: $-\left(\sqrt{-x+1}\right)^2 = -(-x+1) = x - 1$

Sloppy Writing (fractions)

Well, this one is hard to show using typesetting software! Work needs to be written legibly so it can be understood. Also, work needs to be written legibly so errors don't creep into a solution. It is not hard to imagine that without care, it is possible to incorrectly recopy

$$\frac{1}{x+1} \quad \text{as} \quad \frac{1}{x} + 1.$$

Not Checking Your Work

One thing that is not stressed enough is checking your work. Redoing the problem as a check should be done only as a last resort, since if there is a mistake you are liable to make the same mistake twice.

If at all possible, you should check your work by some means that is different than how you solved the problem. And you should check individual steps in a solution as you proceed. The act of checking also provides practice with mathematics that will help you solve more complicated problems in the future.

- If you found the roots of an equation, plug the numbers you found back in and make sure they are the roots.
- If you factor a polynomial, multiply the factors together to make sure you factored correctly.
- If you get the inverse of a function, you can check by showing: $f(f^{-1}(x)) = x$.

Trying to Memorize Instead of Understanding

Some aspects of math need to be memorized, for example logarithm rules and the quadratic formula. But you should work towards understanding the problem, and the general method of constructing a solution. Sometimes I will provide you with something like the "three steps to a solution" (for example, in calculating inverse functions). But most problems in mathematics are complicated, with many parts. It is difficult to see all the steps you will need to take when you begin a problem. However, with practice, you can develop the skills necessary to break large problems into smaller problems, solve the smaller problems, and then put everything back together to solve the larger problem. Remember,

It is not a mistake to write something down incorrectly initially,
but it is a mistake to not go back and correct it!