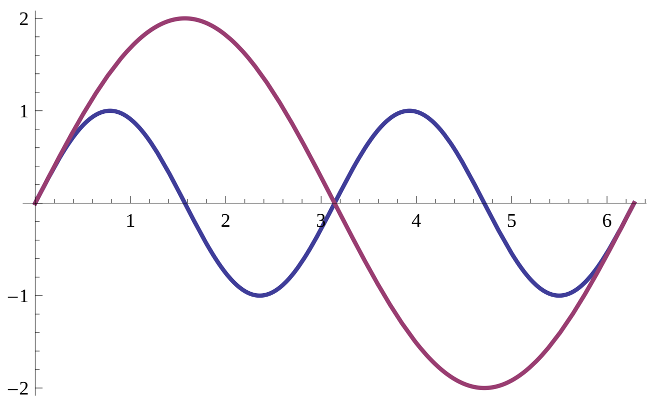
An Example of Something Not an Identity:
$\sin(2x) \neq 2\sin(x)$

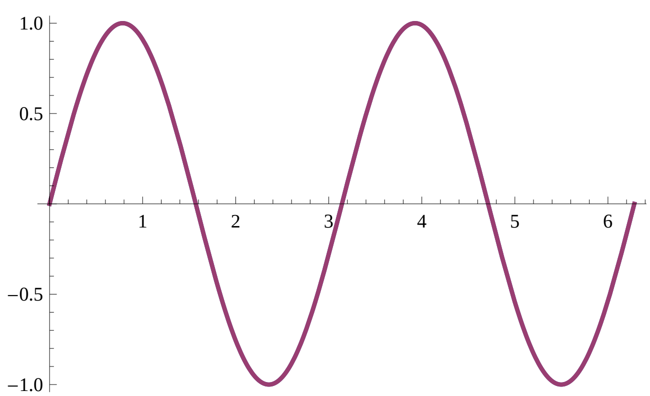
```
Plot[{Sin[2 x], 2 Sin[x]}, {x, 0, 2 Pi}, PlotStyle → Thick]
```



Note: there are particular values of *x* where the two are equal $(0,\pi,2\pi)$, but they are not equal for all values of *x*! Identities must be equal for all values of *x*.

An Example of an Identity:
$\sin(2x) = 2\sin(x)\cos(x)$

```
Plot[{Sin[2 x], 2 Sin[x] Cos[x]}, {x, 0, 2 Pi}, PlotStyle → Thick]
```



This shows that $\sin(2x) = 2\sin(x)\cos(x)$ is a trig identity since it is true for all values of *x* (there are two curves plotted, but they have exactly the same value for all *x* so it looks like only one curve is plotted). We will be deriving trig identities such as this in the coming weeks, and not relying on graphs which don't really prove trig identities.

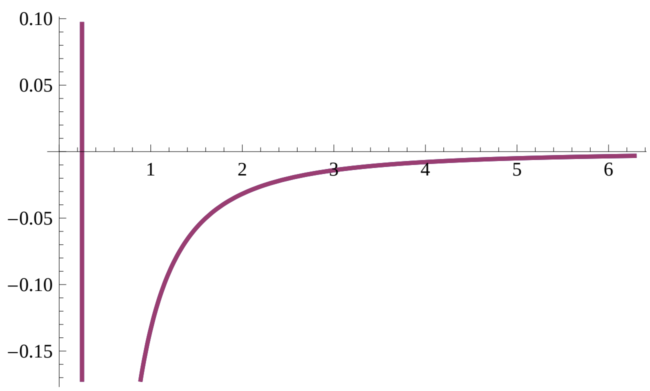
How Trig Identities Are Used

Identities are used to algebraically manipulate the trigonometric functions, so it is important that we know trigonometric identities if we wish to be able to work effectively with trig functions.

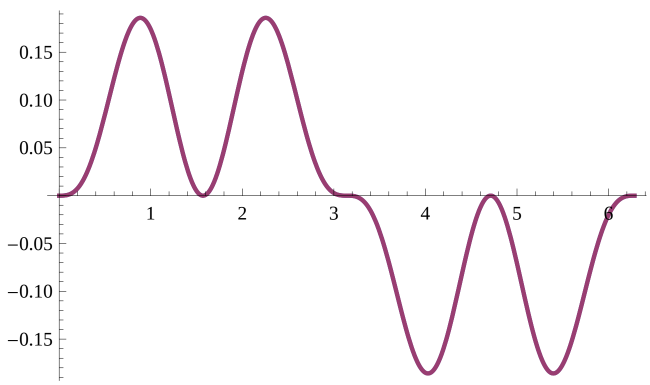
Type of Function	Rules	Example
Rational	Rules of Algebra Common Denominator Factoring, etc	$\frac{1}{4x+1} - \frac{1}{4x-1} = \frac{-2}{16x^2-1}$
Trigonometric	Trig Identities	$\cos^2(x)\sin^3(x) = \sin(x)\left(\cos^2(x) - \cos^4(x)\right)$
Exponential & Logarithmic	Rules of Exponents Rules of Logarithms	$e^{3x} e^{4x} = e^{7x}$

All the examples above are really identities, as the following plots show:

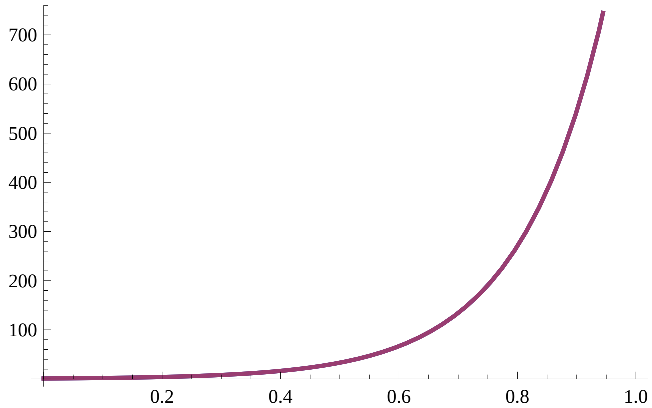
Plot$\left[\left\{\frac{1}{4x+1} - \frac{1}{4x-1}, \frac{-2}{16x^2-1}\right\}, \{x, 0, 2\,\text{Pi}\}, \text{PlotStyle} \to \text{Thick}\right]$



Plot$\left[\left\{\text{Cos}[x]^2\,\text{Sin}[x]^3, \text{Sin}[x]\left(\text{Cos}[x]^2 - \text{Cos}[x]^4\right)\right\}, \{x, 0, 2\,\text{Pi}\}, \text{PlotStyle} \to \text{Thick}\right]$



```
Plot[{Exp[3 x] Exp[4 x], Exp[7 x]}, {x, 0, 1}, PlotStyle → Thick]
```



What Lies Ahead

I will focus on showing us where the important trig identities come from (deriving them), and when we are done I will give you some ideas on how to remember them. I feel there are three keystone trig identities which must be memorized, and the other ones can be quickly found from these three if needed. Often, the best way to remember a trig identity is NOT to go back to how we derived it in the first place!