

Questions

Example Find the derivative of $y = (x^2 + 1)(x^3 + 1)$ in two ways: by using the product rule and by performing the multiplication first. Do your answers agree?

Example Differentiate $\frac{e^x}{1+x}$.

Example Differentiate $f(x) = \frac{ax+b}{cx+d}$.

Solutions

Example Find the derivative of $y = (x^2 + 1)(x^3 + 1)$ in two ways: by using the product rule and by performing the multiplication first. Do your answers agree?

$$\begin{aligned} y &= (x^2 + 1)(x^3 + 1) \\ \frac{dy}{dx} &= \frac{d}{dx}[(x^2 + 1)(x^3 + 1)] \\ &= (x^2 + 1)\frac{d}{dx}[(x^3 + 1)] + (x^3 + 1)\frac{d}{dx}[(x^2 + 1)] \\ &= (x^2 + 1)(3x^2) + (x^3 + 1)(2x) \\ &= (3x^4 + 3x^2) + (2x^4 + 2x) \\ &= 5x^4 + 3x^2 + 2x \end{aligned}$$

$$\begin{aligned} y &= (x^2 + 1)(x^3 + 1) \\ &= x^5 + x^3 + x^2 + 1 \\ \frac{dy}{dx} &= \frac{dy}{dx}[x^5 + x^3 + x^2 + 1] \\ &= 5x^4 + 3x^2 + 2x \end{aligned}$$

The two answers, as expected, agree.

Example Differentiate $\frac{e^x}{1+x}$.

$$\begin{aligned} y &= \frac{e^x}{1+x} \\ \frac{dy}{dx} &= \frac{d}{dx} \left[\frac{e^x}{1+x} \right] \\ &= \frac{(1+x)\frac{d}{dx}[e^x] - e^x\frac{d}{dx}[1+x]}{(1+x)^2} \quad (\text{quotient rule}) \\ &= \frac{(1+x)e^x - e^x[1]}{(1+x)^2} \\ &= \frac{xe^x}{(1+x)^2} \end{aligned}$$

Example Differentiate $f(x) = \frac{ax + b}{cx + d}$.

$$\begin{aligned}f(x) &= \frac{ax + b}{cx + d} \\ \frac{d}{dx} f(x) &= \frac{d}{dx} \left[\frac{ax + b}{cx + d} \right] \\ &= \frac{(cx + d) \frac{d}{dx}(ax + b) - (ax + b) \frac{d}{dx}(cx + d)}{(cx + d)^2} \\ &= \frac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2} \\ &= \frac{(acx + da) - (acx + bc)}{(cx + d)^2} = \frac{da - bc}{(cx + d)^2}\end{aligned}$$