Questions

Example Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

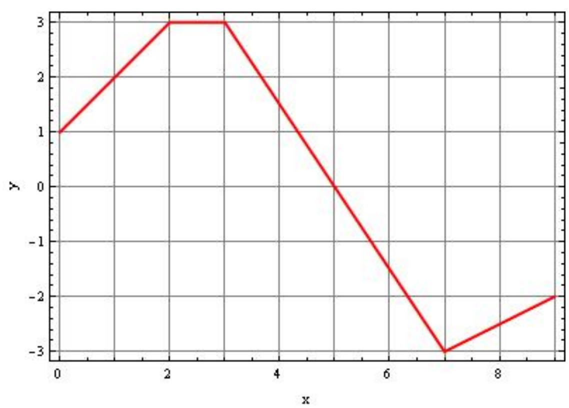
$$\int_1^5 x^2 e^{-x}\,dx, n = 4.$$

Example Express the limit as a definite integral on the given interval.

$$\lim_{n\to\infty}\sum_{i=1}^{n} x_i \sin x_i \Delta x, \quad [0, \pi].$$

Example (5.2.33) The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.

$$(a) \int_0^2 f(x)\,dx \quad (b) \int_0^5 f(x)\,dx \quad (c) \int_5^7 f(x)\,dx \quad (d) \int_0^9 f(x)\,dx$$



Example Evaluate the integral by interpreting it in terms of areas.

$$\int_{-3}^0 (1 + \sqrt{9 - x^2})\,dx.$$

Example Evaluate the integral by interpreting it in terms of areas.

$$\int_{-1}^2 |x|\,dx.$$

Solutions

Example Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

$$\int_1^5 x^2 e^{-x}\, dx, n = 4.$$

The Midpoint Rule is

$$\int_a^b f(x)\, dx \sim \sum_{i=1}^{n} f(\bar{x_i})\Delta x$$

In this case we have $b = 5$, $a = 1$, $n = 4$, so $\Delta x = (b - a)/n = 4/4 = 1$.
$\bar{x_1} = 1.5$, $\bar{x_2} = 2.5$, $\bar{x_3} = 3.5$, $\bar{x_4} = 4.5$.

$$\begin{aligned}
\int_1^5 f(x)\, dx &\sim (f(\bar{x_1}) + f(\bar{x_2}) + f(\bar{x_3}) + f(\bar{x_4}))\Delta x \\
&\sim (f(1.5) + f(2.5) + f(3.5) + f(4.5))(1) \\
\int_1^5 x^2 e^{-x}\, dx &\sim (1.5)^2 e^{-1.5} + (2.5)^2 e^{-2.5} + (3.5)^2 e^{-3.5} + (4.5)^2 e^{-4.5} \\
&= 1.6100
\end{aligned}$$

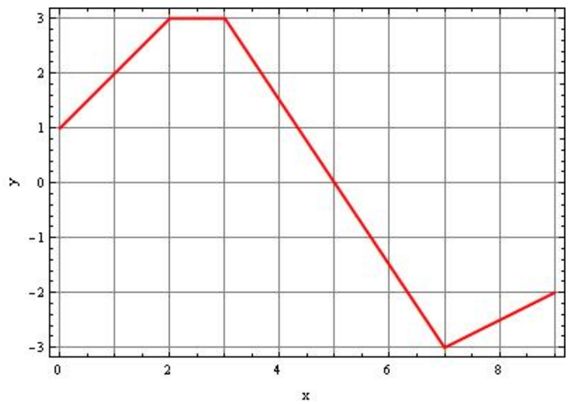
Example Express the limit as a definite integral on the given interval.

$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i \sin x_i \Delta x, \quad [0, \pi].$$

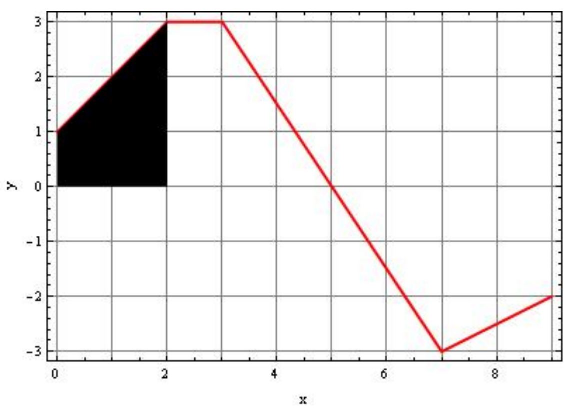
$$\int_0^\pi x \sin x\, dx.$$

Example (5.2.33) The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.

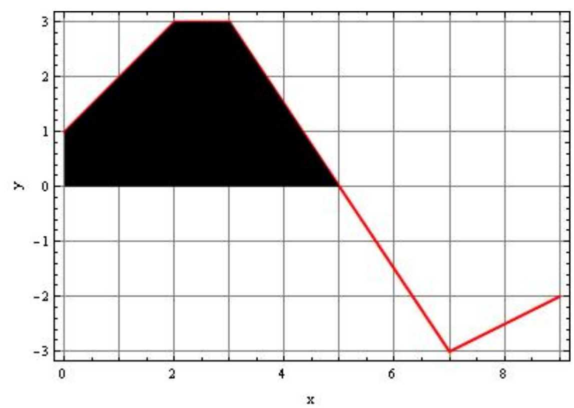
$$(a) \int_0^2 f(x)\, dx \quad (b) \int_0^5 f(x)\, dx \quad (c) \int_5^7 f(x)\, dx \quad (d) \int_0^9 f(x)\, dx$$



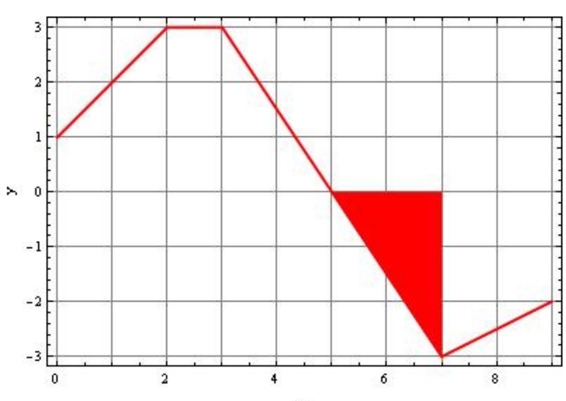
The integrals are represented by the shaded areas. The black areas are above the x axis and so are positive; the red areas are below the x-axis and so are negative.



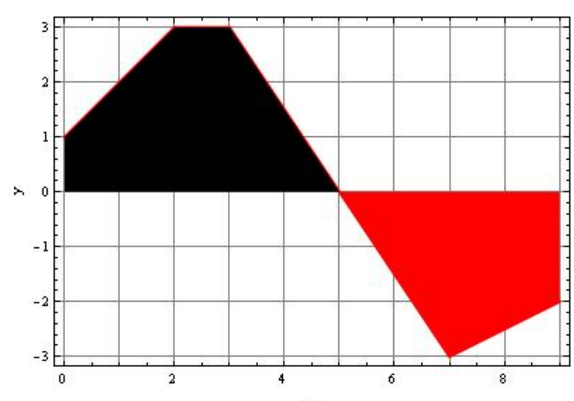
(a) $\int_0^2 f(x)\,dx = 4$



(b) $\int_0^5 f(x)\,dx = 10$



(a) $\int_5^7 f(x)\,dx = -3$



(b) $\int_0^9 f(x)\,dx = 2$

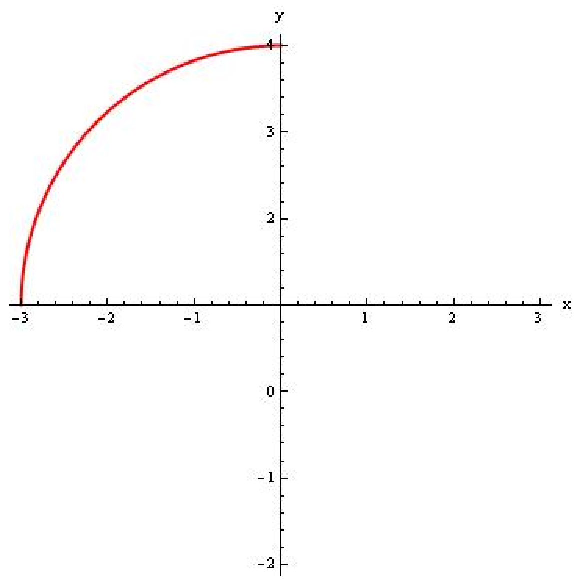
Example Evaluate the integral by interpreting it in terms of areas.

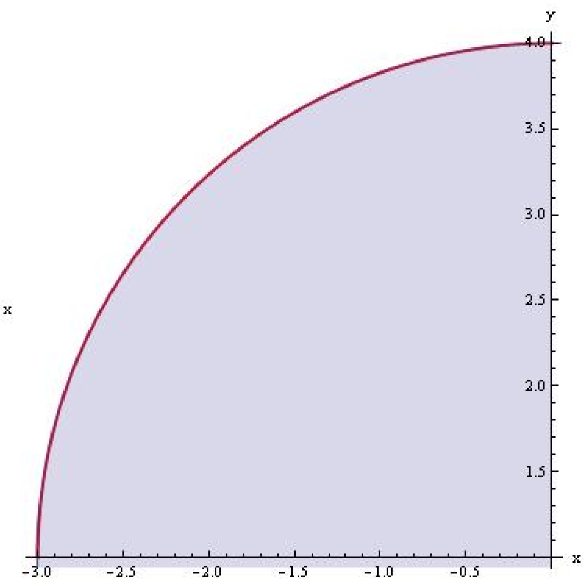
$$\int_{-3}^{0} (1 + \sqrt{9 - x^2})\,dx.$$

First, we need to know what the region looks like. we have:

$$\begin{aligned} y &= 1 + \sqrt{9 - x^2} \\ (y-1)^2 &= 9 - x^2 \\ (y-1)^2 + x^2 &= 3^2 \end{aligned}$$

which is a circle of radius 3 and center $(0, 1)$. Now we can sketch the region:





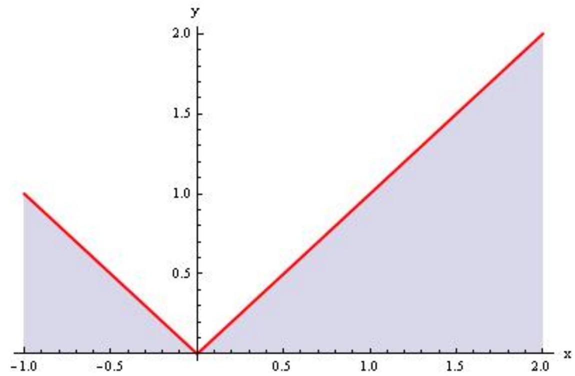
The integral is given by the sum of the two shaded regions, the light blue region is a quarter of the area of a circle of radius 3, and the gray region is a rectangle of length 3 and height 1. Therefore,

$$\int_{-3}^{0} (1 + \sqrt{9 - x^2})\, dx = \frac{1}{4}\pi(3)^2 + (3)(1) = \frac{9}{4}\pi + 3.$$

Example Evaluate the integral by interpreting it in terms of areas.

$$\int_{-1}^{2} |x|\, dx.$$

Here is the region, which is two triangles:



$$\int_{-1}^{2} |x|\, dx = \frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) = \frac{5}{2}.$$