Questions

Example Evaluate the integral by making the given substitution.

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx, \quad u=\sqrt{x}.$$

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$$\int e^{\sin\theta}\cos\theta\,d\theta, \quad u=\sin\theta.$$

Example Evaluate the indefinite integral

$$\int \frac{ax+b}{\sqrt{ax^2+2bx+c}}\,dx.$$

Example Evaluate the definite integral, if it exists.

$$\int_0^7 \sqrt{4+3x}\,dx$$

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$$\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$$

Example Evaluate the definite integral, if it exists.

$$\int_{1/6}^{1/2} \csc(\pi t)\cot(\pi t)\,dt$$

Solutions

Example Evaluate the integral by making the given substitution.

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx, \quad u=\sqrt{x}.$$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx \quad = \quad 2\int \sin\sqrt{x}\,\left(\frac{1}{2\sqrt{x}}\,dx\right) \qquad \text{Substitution: } u=\sqrt{x},\ du=\frac{1}{2\sqrt{x}}dx$$

$$\begin{aligned} &= 2\int \sin u \ (du) \\ &= 2\int \sin u \, du \\ &= -2\cos u + c \\ &= -2\cos\sqrt{x} + c \end{aligned}$$

Example Evaluate the integral by making the given substitution.

$$\int e^{\sin\theta}\cos\theta \, d\theta, \quad u = \sin\theta.$$

$$\begin{aligned} \int e^{\sin\theta}\cos\theta \, d\theta &= \int e^{\sin\theta}(\cos\theta \, d\theta) \\ &\qquad \text{Substitution: } u = \sin\theta, \ du = \cos\theta d\theta \\ &= \int e^u (du) \\ &= \int e^u \, du \\ &= e^u + c \\ &= e^{\sin\theta} + c \end{aligned}$$

Example Evaluate the indefinite integral

$$\int \frac{ax+b}{\sqrt{ax^2+2bx+c}} \, dx.$$

$$\begin{aligned} \int \frac{ax+b}{\sqrt{ax^2+2bx+c}} \, dx &= \frac{1}{2}\int \frac{1}{\sqrt{ax^2+2bx+c}} \left((2ax+2b)dx\right) \\ &\qquad \text{Substitution: } u = ax^2+2bx+c, \ du = (2ax+2b)dx \\ &= \frac{1}{2}\int \frac{1}{\sqrt{u}} \, (du) \\ &= \frac{1}{2}\int u^{-1/2} \, du \\ &= \frac{1}{2}\cdot\frac{1}{1/2}u^{1/2} + k \\ &= \sqrt{ax^2+bx+c} + k \end{aligned}$$

Example Evaluate the definite integral, if it exists.

$$\int_0^7 \sqrt{4+3x} \, dx$$

$$\int_0^7 \sqrt{4+3x}\,dx \qquad \text{Substitution} \quad \begin{array}{ll} u = 4+3x & \text{when } x=0, u=4 \\ du = 3\,dx & \text{when } x=7, u=25 \end{array}$$

$$\begin{aligned}
&= \frac{1}{3}\int_4^{25} \sqrt{u}\,du \\
&= \frac{1}{3}\int_4^{25} u^{1/2}\,du \\
&= \frac{1}{3}\cdot\frac{1}{3/2}u^{3/2}\Big|_4^{25} \\
&= \frac{2}{9}((25)^{3/2} - (4)^{3/2}) \\
&= \frac{2}{9}(125-8) \\
&= \frac{234}{9} = 26
\end{aligned}$$

Example Evaluate the definite integral, if it exists.

$$\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$$

$$\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx \qquad \text{Substitution} \quad \begin{array}{ll} u = x^2 & \text{when } x=0, u=0 \\ du = 2x\,dx & \text{when } x=\sqrt{\pi}, u=\pi \end{array}$$

$$\begin{aligned}
&= \frac{1}{2}\int_0^{\sqrt{\pi}} \cos(x^2)\,(2xdx) \\
&= \frac{1}{2}\int_0^{\pi} \cos(u)\,du \\
&= \frac{1}{2}\sin u|_0^{\pi} \\
&= \frac{1}{2}(\sin\pi - \sin 0) \\
&= \frac{1}{2}(0-0) = 0
\end{aligned}$$

Example Evaluate the definite integral, if it exists.

$$\int_{1/6}^{1/2} \csc(\pi t)\cot(\pi t)\,dt$$

$$\int_{1/6}^{1/2} \csc(\pi t)\cot(\pi t)\,dt \quad = \quad \int_{1/6}^{1/2} \frac{1}{\sin(\pi t)}\frac{\cos(\pi t)}{\sin(\pi t)}\,dt$$

$$
\begin{aligned}
&= \int_{1/6}^{1/2} \sin^{-2}(\pi t)\cos(\pi t)\,dt \\
&\quad \text{Substitution } \begin{array}{ll} u = \sin(\pi t) & \text{when } t = 1/2, u = 1 \\ du = \pi\cos(\pi t)\,dt & \text{when } t = 1/6, u = 1/2 \end{array} \\
&= \frac{1}{\pi}\int_{1/6}^{1/2} \sin^{-2}(\pi t)(\pi\cos(\pi t)\,dt) \\
&= \frac{1}{\pi}\int_{1/2}^{1} u^{-2}\,du \\
&= -\frac{1}{\pi}u^{-1}|_{1/2}^{1} \\
&= -\frac{1}{\pi}(1-2) = \frac{1}{\pi}
\end{aligned}
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