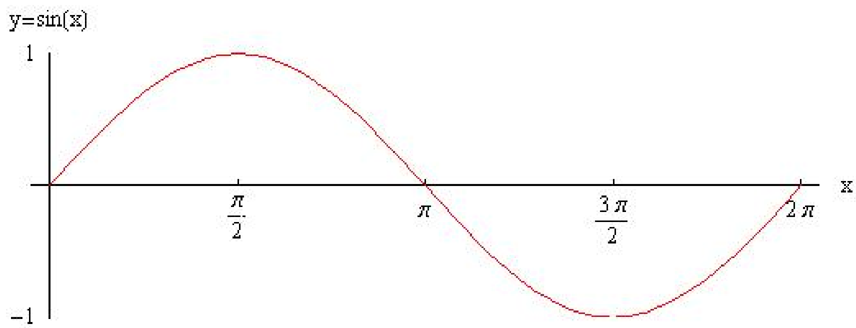
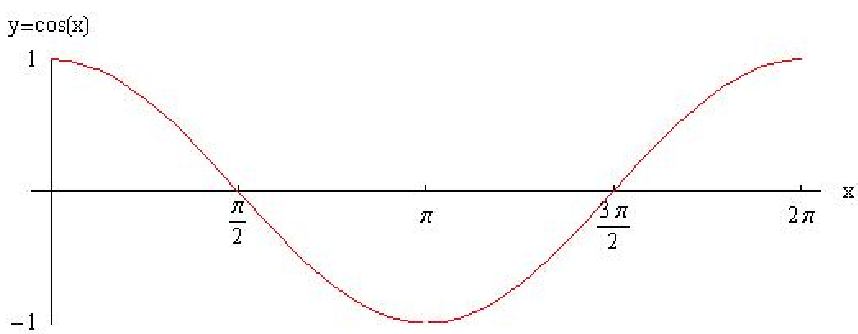
Appendix D has a trigonometric review. This material is meant to outline some of the proofs of identities, help you remember the values of the trig functions at special values, and help you see how the trig identities are related. You will not be tested on this material directly; you mainly need to have certain trig identities memorized, or know how to derive them if you need them. Remember–memorized means memorized correctly, not just that you are familiar with something! If you use an identity in class or on the homework that means it is important and might show up again.

The Sine Function $y = \sin x$



Domain: $x \in \mathbb{R}$
Range: $y \in [-1, 1]$
Continuity: continuous for all x
Increasing-decreasing behaviour: alternately increasing and decreasing
Symmetry: odd ($\sin(-x) = -\sin(x)$))
Boundedness: bounded above and below
Local Extrema: absolute max of $y = 1$, absolute min of $y = -1$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $+1$ and -1.
This is a periodic function with period 2π.

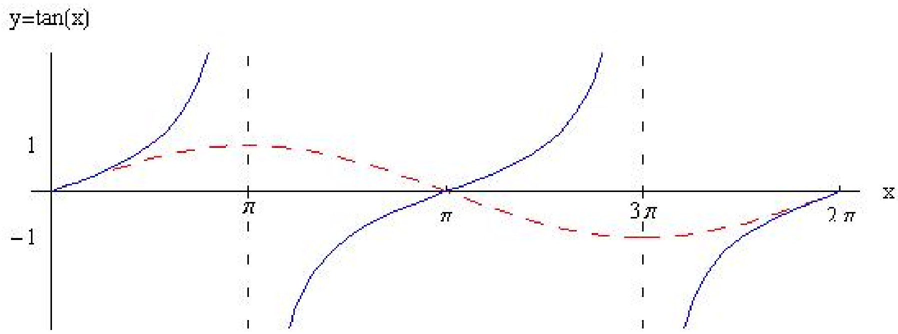
The Cosine Function $y = \cos x$



Domain: $x \in \mathbb{R}$
Range: $y \in [-1, 1]$
Continuity: continuous for all x
Increasing-decreasing behaviour: alternately increasing and decreasing

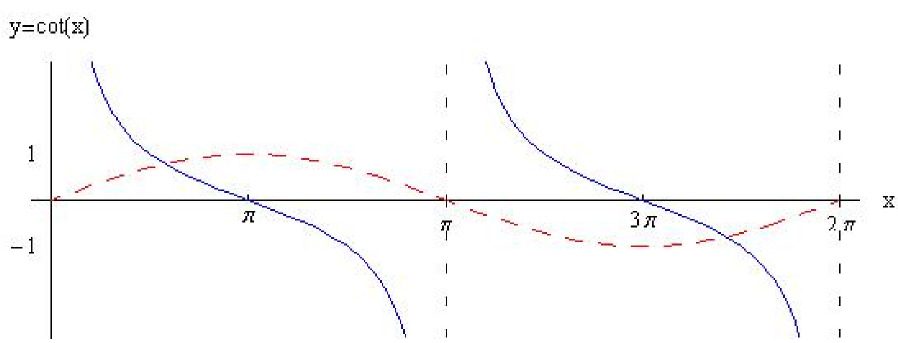
Symmetry: even ($\cos(-x) = \cos(x)$))
Boundedness: bounded above and below
Local Extrema: absolute max of $y = 1$, absolute min of $y = -1$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $+1$ and -1.
This is a periodic function with period 2π.

The Tangent Function $y = \tan x = \dfrac{\sin x}{\cos x}$



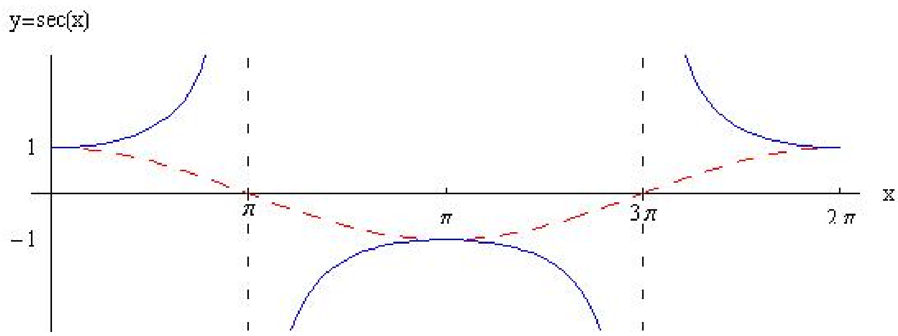
Domain: $x \in \mathbb{R}$ except $x = \dfrac{\pi}{2} + k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
Range: $y \in \mathbb{R}$
Continuity: continuous on its domain
Increasing-decreasing behaviour: increasing on each interval in its domain
Symmetry: odd ($\tan(-x) = -\tan(x)$))
Boundedness: not bounded
Local Extrema: none
Horizontal Asymptotes: none
Vertical Asymptotes: $x = \dfrac{\pi}{2} + k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period π.

The Cotangent Function $y = \cot x = \dfrac{\cos x}{\sin x}$



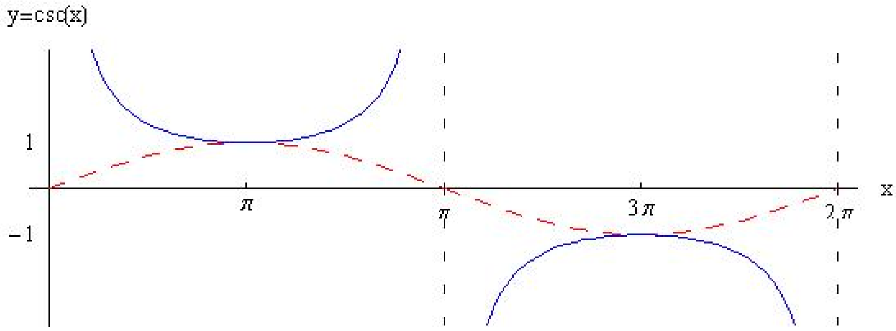
Domain: $x \in \mathbb{R}$ except $x = k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
Range: $y \in \mathbb{R}$
Continuity: continuous on its domain
Increasing-decreasing behaviour: decreasing on each interval in its domain
Symmetry: odd ($\cot(-x) = -\cot(x)$))
Boundedness: not bounded
Local Extrema: none
Horizontal Asymptotes: none
Vertical Asymptotes: $x = k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period π.

The Secant Function $y = \sec x = \dfrac{1}{\cos x}$



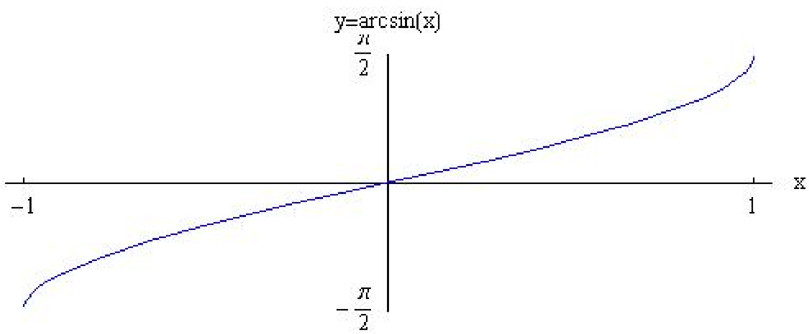
Domain: $x \in \mathbb{R}$ except $x = \dfrac{\pi}{2} + k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
Range: $y \in (-\infty, -1] \cup [1, \infty)$
Continuity: continuous on its domain
Increasing-decreasing behaviour: increases and decreases on each interval in its domain
Symmetry: even ($\sec(-x) = \sec(x)$))
Boundedness: not bounded
Local Extrema: local min at $x = 2k\pi$, local max at $x = (2k+1)\pi$, $k = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
Horizontal Asymptotes: none
Vertical Asymptotes: $x = \dfrac{\pi}{2} + k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period 2π.

The Cosecant Function $y = \csc x = \dfrac{1}{\sin x}$



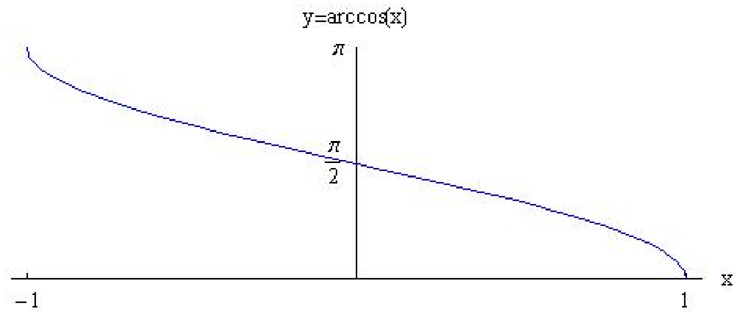
Domain: $x \in \mathbb{R}$ except $x = k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
Range: $y \in (-\infty, -1] \cup [1, \infty)$
Continuity: continuous on its domain
Increasing-decreasing behaviour: increases and decreases on each interval in its domain
Symmetry: odd $(\csc(-x) = -\csc(x)))$
Boundedness: not bounded
Local Extrema: local min at $x = \pi/2 + 2k\pi$, local max at $x = 3\pi/2 + 2k\pi$, $k = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
Horizontal Asymptotes: none
Vertical Asymptotes: $x = k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period 2π.

The Inverse Sine Function $y = \arcsin x$



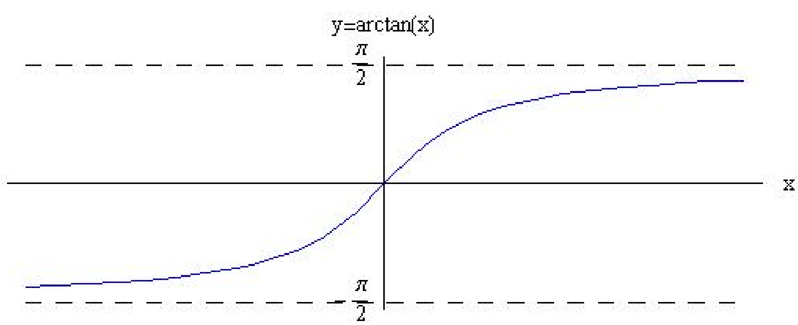
Domain: $x \in [-1, 1]$
Range: $y \in [-\pi/2, \pi/2]$
Continuity: continuous for all x in domain
Increasing-decreasing behaviour: increasing
Symmetry: odd $(\arcsin(-x) = -\arcsin(x)))$
Boundedness: bounded above and below
Local Extrema: absolute max of $y = \pi/2$, absolute min of $y = -\pi/2$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: The limits as x approaches $\pm\infty$ do not exist.

The Inverse Cosine Function $y = \arccos x$



Domain: $x \in [-1, 1]$
Range: $y \in [0, \pi]$
Continuity: continuous for all x in domain
Increasing-decreasing behaviour: decreasing
Symmetry: none
Boundedness: bounded above and below
Local Extrema: absolute max of $y = \pi$, absolute min of $y = 0$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: The limits as x approaches $\pm\infty$ do not exist.

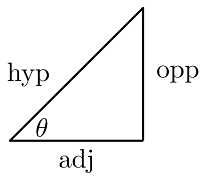
The Inverse Tangent Function $y = \arctan x$



Domain: $x \in \mathbb{R}$
Range: $y \in (-\pi/2, \pi/2)$
Continuity: continuous for all x
Increasing-decreasing behaviour: increasing
Symmetry: odd ($\arctan(-x) = -\arctan(x)$))
Boundedness: bounded above and below
Local Extrema: absolute max of $y = \pi/2$, absolute min of $y = -\pi/2$
Horizontal Asymptotes: $y = \pm\pi/2$
Vertical Asymptotes: none
End behaviour: $\lim_{x\to\infty} \arctan x = \dfrac{\pi}{2}$ $\qquad \lim_{x\to-\infty} \arctan x = -\dfrac{\pi}{2}$

• Notation: $\arcsin x = \sin^{-1} x \neq (\sin x)^{-1} = \dfrac{1}{\sin x}$ and similarly for $\arccos x$ and $\arctan x$.

Right Angle Triangles

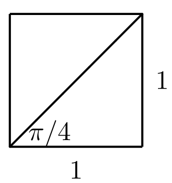


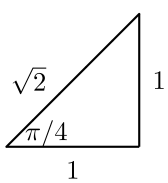
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \quad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

The six basic trigonometric functions relate the angle θ to ratios of the length of the sides of the right triangle. For certain triangles, the trig functions of the angles can be found geometrically. These special triangles occur frequently enough that it is expected that you learn the value of the trig functions for the special angles.

A 45-45-90 Triangle

Consider the square given below.





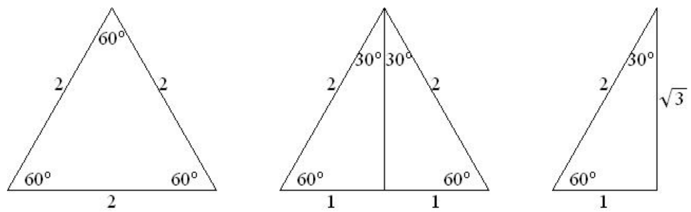
The angle here must be $\pi/4$ radians, since this triangle is half of a square of side length 1.

Now, we can write down all the trig functions for an angle of $\pi/4$ radians = 45 degrees:

$$\sin\left(\frac{\pi}{4}\right) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \qquad \csc\left(\frac{\pi}{4}\right) = \frac{1}{\sin\left(\frac{\pi}{4}\right)} = \sqrt{2}$$
$$\cos\left(\frac{\pi}{4}\right) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \qquad \sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \sqrt{2}$$
$$\tan\left(\frac{\pi}{4}\right) = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{1} = 1 \qquad \cot\left(\frac{\pi}{4}\right) = \frac{1}{\tan\left(\frac{\pi}{4}\right)} = 1$$

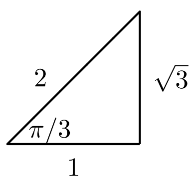
A 30-60-90 Triangle

Consider the equilateral triangle given below. Geometry allows us to construct a 30-60-90 triangle:



We can now determine the six trigonometric functions at two more angles!

$60^{\text{o}} = \dfrac{\pi}{3}$ radians:

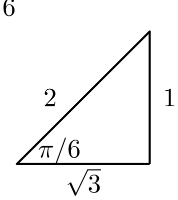


$$\sin\left(\frac{\pi}{3}\right) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \qquad \csc\left(\frac{\pi}{3}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = \frac{2}{\sqrt{3}}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{2} \qquad \sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = 2$$

$$\tan\left(\frac{\pi}{3}\right) = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sqrt{3}}{1} = \sqrt{3} \qquad \cot\left(\frac{\pi}{3}\right) = \frac{1}{\tan\left(\frac{\pi}{3}\right)} = \frac{1}{\sqrt{3}}$$

$30^{\text{o}} = \dfrac{\pi}{6}$ radians:

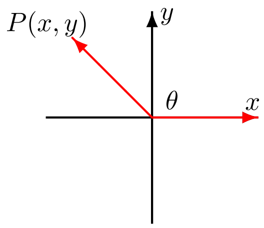


$$\sin\left(\frac{\pi}{6}\right) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} \qquad \csc\left(\frac{\pi}{6}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = 2$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \qquad \sec\left(\frac{\pi}{6}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{2}{\sqrt{3}}$$

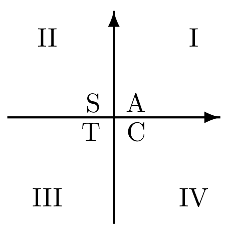
$$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{\sqrt{3}} \qquad \cot\left(\frac{\pi}{6}\right) = \frac{1}{\tan\left(\frac{\pi}{3}\right)} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Obtuse Angles



If we label the point at the end of the terminal side as $P(x, y)$, and if we let $r = \sqrt{x^2 + y^2}$, we can construct the following relationships between the six trig functions and our diagram:

$$\cos\theta = \frac{x}{r}, \qquad \sin\theta = \frac{y}{r}, \qquad \tan\theta = \frac{y}{x},\ x \neq 0$$
$$\csc\theta = \frac{r}{y},\ y \neq 0, \qquad \sec\theta = \frac{r}{x},\ x \neq 0, \qquad \cot\theta = \frac{x}{y},\ y \neq 0$$



The CAST diagram tells us the sign of sine, cosine and tangent in the quadrants.

Quadrant IV: Cosine is positive, the other two are negative.
Quadrant I: All are positive.
Quadrant II: Sine is positive, the other two are negative.
Quadrant III: Tangent is positive, the other two are negative.

Identities

You will need to be able to know the basic trig identities, or derive them. I recommend memorizing a few, and deriving others that you will need when necessary. I would memorize $\cos^2 x + \sin^2 x = 1$, $\cos(u - v) = \cos u \cos v + \sin u \sin v$, $\sin(u + v) = \sin u \cos v + \cos u \sin v$.

- From the definition of the trig functions:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

- Pythagorean Identities:

$$\cos^2\theta + \sin^2\theta = 1$$

Divide by $\cos^2\theta$:

$$\begin{aligned} \frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} &= \frac{1}{\cos^2\theta} \\ 1 + \tan^2\theta &= \sec^2\theta \end{aligned}$$

Divide by $\sin^2\theta$:

$$\begin{aligned} \frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} &= \frac{1}{\sin^2\theta} \\ \cot^2\theta + 1 &= \csc^2\theta \end{aligned}$$

- Cofunction Identities

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \qquad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right) \qquad \tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right) \qquad \sec\theta = \csc\left(\frac{\pi}{2} - \theta\right) \qquad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

- Even/Odd Identities

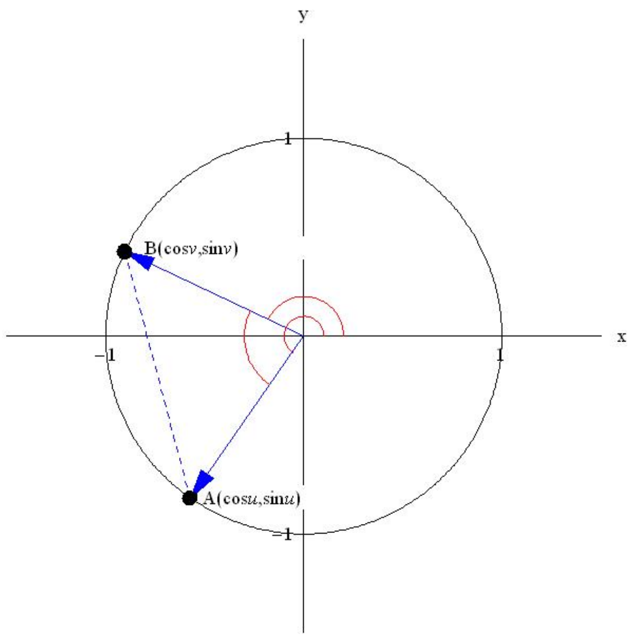
$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

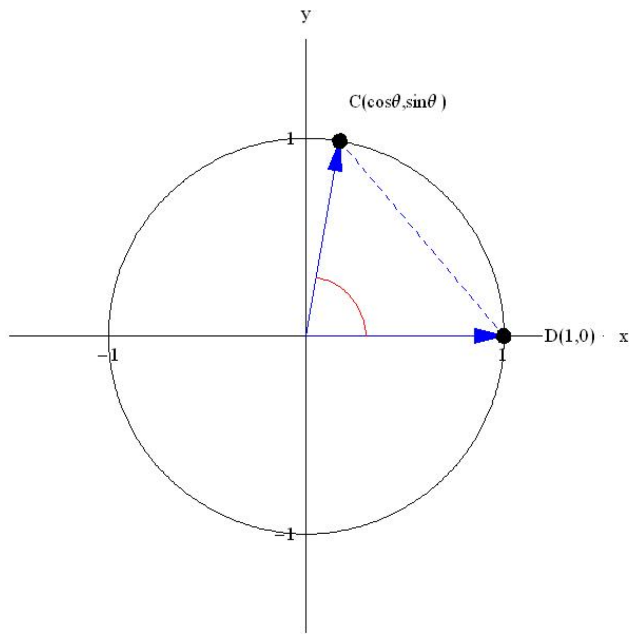
$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

- The Cosine of a Difference Identity Derivation (for your information)

To get the cosine of a difference, let's draw a diagram involving the unit circle and see what we can learn.

The angle u leads to a point $A(\cos u, \sin u)$ on the unit circle.
The angle v leads to a point $B(\cos v, \sin v)$ on the unit circle.
The angle $\theta = u - v$ is the angle between the the terminal sides of u and v.
The dotted line connects the points A and B.





We can rotate the geometry of this picture so that the angle θ is in standard position.

The dashed lines are the same length in both pictures. Therefore, we can use the distance between two points formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$ (see page 16)

and we can write:

$$\sqrt{(\cos u - \cos v)^2 + (\sin u - \sin v)^2} = \sqrt{(\cos\theta - 1)^2 + (\sin\theta - 0)^2}$$

Now all we have to do is simplify this expression! Remember, $\theta = u - v$, so we want to solve this for $\cos\theta = \cos(u - v)$.

$$\begin{aligned}
(\sqrt{(\cos u - \cos v)^2 + (\sin u - \sin v)^2})^2 &= (\sqrt{(\cos\theta - 1)^2 + (\sin\theta - 0)^2})^2 \\
(\cos u - \cos v)^2 + (\sin u - \sin v)^2 &= (\cos\theta - 1)^2 + (\sin\theta - 0)^2 \\
(\cos^2 u + \cos^2 v - 2\cos u\cos v) + (\sin^2 u + \sin^2 v - 2\sin u\sin v) &= (\cos^2\theta + 1 - 2\cos\theta) + \sin^2\theta \\
(\cos^2 u + \sin^2 u) - 2\cos u\cos v + (\cos^2 v + \sin^2 v) - 2\sin u\sin v &= (\cos^2\theta + \sin^2\theta) + 1 - 2\cos\theta) \\
(1) - 2\cos u\cos v + (1) - 2\sin u\sin v &= (1) + 1 - 2\cos\theta \\
2 - 2\cos u\cos v - 2\sin u\sin v &= 2 - 2\cos\theta \\
2 - 2\cos u\cos v - 2\sin u\sin v &= 2 - 2\cos\theta \\
-2\cos u\cos v - 2\sin u\sin v &= -2\cos\theta \\
+\cos u\cos v + \sin u\sin v &= +\cos\theta \\
\cos\theta = \cos(u - v) &= \cos u\cos v + \sin u\sin v
\end{aligned}$$

We have arrived at the trig identity $\cos(u - v) = \cos u\cos v + \sin u\sin v$.

- The Cosine of a Sum Identity $\cos(u+v) = \cos u \cos v - \sin u \sin v$.

- The Sine of a Sum/Difference Identities $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$.

- The Tangent of a Difference or Sum Identities $\tan(u \pm v) = \dfrac{\sin(u \pm v)}{\cos(u \pm v)} = \dfrac{\sin u \cos v \pm \cos u \sin v}{\cos u \cos v \mp \sin u \sin v}$.

- The double angle identities are found from letting $u = v$ in the sum identities.

$$
\begin{aligned}
\cos(u+v) &= \cos u \cos v - \sin u \sin v \\
\cos(2u) = \cos(u+u) &= \cos u \cos u - \sin u \sin u \\
&= \cos^2 u - \sin^2 u \\
&= \cos^2 u - (1 - \cos^2 u) \\
&= 2\cos^2 u - 1 \\
&= 2(1 - \sin^2 u) - 1 \\
&= 1 - 2\sin^2 u
\end{aligned}
$$

$$
\begin{aligned}
\sin(u+v) &= \sin u \cos v + \cos u \sin v \\
\sin(2u) = \sin(u+v) &= \sin u \cos u + \cos u \sin u \\
&= 2 \sin u \cos u
\end{aligned}
$$

$$
\begin{aligned}
\tan(2u) = \frac{\sin(2u)}{\cos(2u)} &= \frac{2\sin u \cos u}{\cos^2 u - \sin^2 u} \\
&= \frac{2\sin u \cos u}{\cos^2 u - \sin^2 u} \cdot (1) \\
&= \frac{2\sin u \cos u}{\cos^2 u - \sin^2 u} \cdot \left(\frac{\left(\frac{1}{\cos^2 u}\right)}{\left(\frac{1}{\cos^2 u}\right)} \right) \\
&= \frac{2\frac{\sin u}{\cos u}}{\frac{\cos^2 u}{\cos^2 u} - \frac{\sin^2 u}{\cos^2 u}} \\
&= \frac{2\tan u}{1 - \tan^2 u}
\end{aligned}
$$

- Power Reducing Identities

The power reducing identities are found by rearranging the double angle identities.

$$
\begin{aligned}
\cos(2u) &= 2\cos^2 u - 1 \\
\cos^2 u &= \frac{1 + \cos 2u}{2}
\end{aligned}
$$

$$
\begin{aligned}
\cos(2u) &= 1 - 2\sin^2 u \\
\sin^2 u &= \frac{1 - \cos 2u}{2}
\end{aligned}
$$

$$
\tan^2 u = \frac{\sin^2 u}{\cos^2 u} = \frac{\left(\frac{1 - \cos 2u}{2}\right)}{\left(\frac{1 + \cos 2u}{2}\right)}
$$

$$
\begin{aligned}
&= \frac{\left(\frac{1-\cos 2u}{2}\right)}{\left(\frac{1+\cos 2u}{2}\right)} \cdot \left(\frac{2}{2}\right) \\
&= \frac{1-\cos 2u}{1+\cos 2u}
\end{aligned}
$$

• Half Angle Identities

The half angle identities are found from the power reducing identities. They have an inherent ambiguity in the sign of the square root, and this ambiguity can only be removed by checking which quadrant $u/2$ lies in on a case-by-case basis.

$$
\begin{aligned}
\cos^2 u &= \frac{1+\cos 2u}{2} \\
\cos^2(u/2) &= \frac{1+\cos u}{2} \\
\cos(u/2) &= \pm\sqrt{\frac{1+\cos u}{2}}
\end{aligned}
$$

$$
\begin{aligned}
\sin^2 u &= \frac{1-\cos 2u}{2} \\
\sin^2(u/2) &= \frac{1-\cos u}{2} \\
\sin(u/2) &= \pm\sqrt{\frac{1-\cos u}{2}}
\end{aligned}
$$

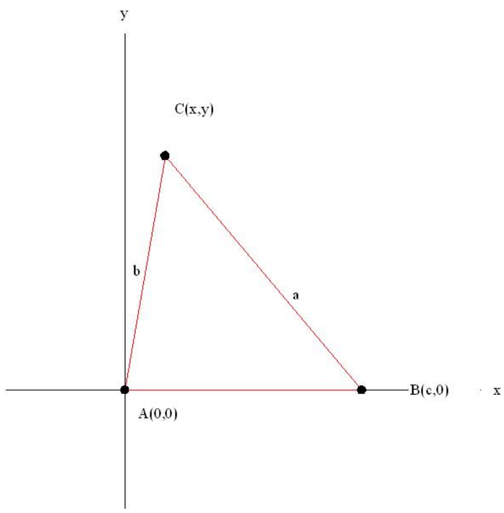
$$
\begin{aligned}
\tan^2 u &= \frac{1-\cos 2u}{1+\cos 2u} \\
\tan^2(u/2) &= \frac{1-\cos u}{1+\cos u} \\
\tan(u/2) &= \pm\sqrt{\frac{1-\cos u}{1+\cos u}}
\end{aligned}
$$

For the half angle tangent identities, we can write two additional identities that do not have the ambiguity of the sign of the square root since the $\sin x$ and $\tan(x/2)$ are both negative in the same intervals.

$$
\begin{aligned}
\tan(u/2) &= \pm\sqrt{\frac{1-\cos u}{1+\cos u}} \\
&= \pm\sqrt{\frac{(1-\cos u)(1-\cos u)}{(1+\cos u)(1-\cos u)}} \\
&= \pm\sqrt{\frac{(1-\cos u)^2}{(1-\cos^2 u)}} \\
&= \pm\sqrt{\frac{(1-\cos u)^2}{\sin^2 u}} = \frac{1-\cos u}{\sin u} \\
\tan(u/2) &= \frac{1-\cos u}{\sin u} \cdot \left(\frac{1+\cos u}{1+\cos u}\right) \\
&= \frac{(1-\cos u)(1+\cos u)}{\sin u(1+\cos u)} = \frac{1-\cos^2 u}{\sin u(1+\cos u)} = \frac{\sin^2 u}{\sin u(1+\cos u)} = \frac{\sin u}{1+\cos u}
\end{aligned}
$$

• Law of Cosines

The law of cosines is a generalization of the Pythagorean theorem. It can be derived in a manner similar to how we derived the identity for $\cos(u - v)$.

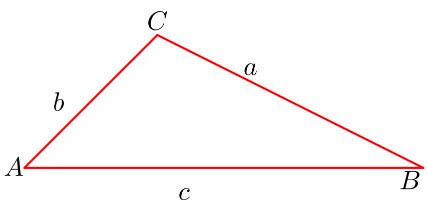


The coordinates of the point C satisfy: $\cos A = \dfrac{x}{b}$ and $\sin A = \dfrac{y}{b}$.

Therefore, $x = b\cos A$ and $y = b\sin A$. Using the distance formula, we can write

$$\begin{aligned}
a &= \sqrt{(x-c)^2 + (y-0)^2} \\
a^2 &= (x-c)^2 + y^2 \\
a^2 &= (b\cos A - c)^2 + (b\sin A)^2 \\
a^2 &= b^2\cos^2 A + c^2 - 2bc\cos A + b^2\sin^2 A \\
a^2 &= b^2(\cos^2 A + \sin^2 A) + c^2 - 2bc\cos A \\
a^2 &= b^2(1) + c^2 - 2bc\cos A \\
a^2 &= b^2 + c^2 - 2bc\cos A
\end{aligned}$$

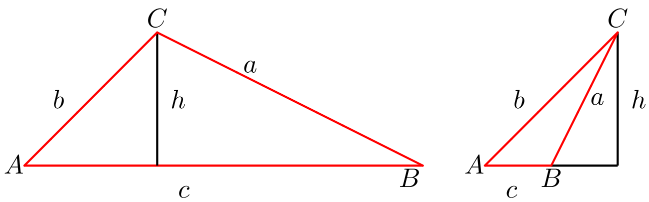
Using a similar technique, you can prove the other law of cosines results.



$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$

- The Law of Sines

There are two possibilities for the shape of the triangle created with interior angles A, B, C and sides of length a, b, c. The sides are labelled opposite their corresponding angles. The perpendicular height is labelled h in both cases.



From either of the diagrams above, we have $\sin A = \dfrac{h}{b}$.

Also, from the diagram on the left, we have $\sin B = \dfrac{h}{a}$.

Also, from the diagram on the right, we have $\sin(\pi - B) = \dfrac{h}{a}$.

$$\begin{aligned}
\sin(u-v) &= \sin u \cos v - \cos u \sin v \\
\sin(\pi - B) &= \sin \pi \cos B - \cos \pi \sin B \\
&= (0) \cos B - (-1) \sin B \\
&= \sin B = \frac{h}{a}
\end{aligned}$$

Therefore, for both triangles we have

$$\begin{aligned}
h = b \sin A &= a \sin B \\
\frac{\sin A}{a} &= \frac{\sin B}{b}
\end{aligned}$$

You could do exactly the same thing where you drop the perpendicular to the other two sides.

This leads to the Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$