MATH 2101 (Ng/Fall 2012)
Examination 2
Friday, November 2, 2012.
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Instructions:

1. Read each question very carefully. Write your name on all answer sheets.
2. Write answers on the space provided on your test paper only; nothing else will be graded.
3. Write your answer(s) to each question **at the space provided for that question.** For instance, **PLEASE DO NOT WRITE YOUR ANSWERS TO PROBLEM 3(B) AT THE LOCATION OF PROBLEM 3(C).**
4. Only **legible** and **mathematically correct** work will be awarded credit.
5. Whenever computation is appropriate, show intermediate steps. Where instructions call for computation of something via a specified method, no credit will be given for solutions derived by other means.
6. This is a **closed book and notes** test; no consultation with anyone but yourself.

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1. (8pts.) Let $\vec{r}(t) = \sin(t)\vec{i} + \cos(t)\vec{j} + 2t\vec{k}$ be the position vector of a moving object at time t, $t \geq 0$. What is the length of the path travelled by the object between $t = 0$ and $t = 2\pi$?

$$L = \int_0^{2\pi} |\vec{r}\,'(t)|\,dt = \int_0^{2\pi} \sqrt{\cos^2 t + \sin^2 t + 4}\ dt \qquad \vec{r}\,'(t) = \langle \cos t, -\sin t, 2\rangle$$

$$= \int_0^{2\pi} \sqrt{5}\ dt$$

$$= \sqrt{5}\, t \Big|_0^{2\pi}$$

$$= 2\sqrt{5}\,\pi$$

2. (8pts.) Let $f(x, y, z) = e^x \cos(y) + e^y cos(x) + z$.

Find the directional derivative of f, at the point $(0, 0, 1)$, in the direction of **V**$=\langle 1, 2, 2 \rangle$.

1st need a unit vector, $\vec{u}$, in the direction of $\vec{V}$ i.e. $\vec{u} = \frac{1}{\sqrt{9}}\vec{V} = \frac{1}{3}\langle 1,2,2\rangle$

2nd $D_{\vec{u}} f = \vec{\nabla} f \bullet \vec{u} = \langle f_x, f_y, f_z \rangle \bullet \vec{u}$

$$= \langle e^x \cos y - e^y \sin x,\ -e^x \sin y + e^y \cos x,\ 1 \rangle \bullet \vec{u}$$

So, at $(0,0,1)$,

$$D_{\vec{u}} f(0,0,1) = \vec{\nabla} f(0,0,1) \bullet \vec{u} = \langle e^0 \cos 0 - e^0 \sin 0,\ -e^0 \sin 0 + e^0 \cos 0,\ 1 \rangle \bullet \vec{u}$$

$$= \langle 1, 1, 1 \rangle \bullet \langle \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{2}{3} \rangle$$

$$= \tfrac{1}{3} + \tfrac{2}{3} + \tfrac{2}{3} = \tfrac{5}{3}$$