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3. (7pts.) Does

$$\lim_{(x,y)\to(0,0)} \frac{y^2 - x^2}{x + y}$$

exist? **Justify your answer.**

Let $f(x,y) = \dfrac{y^2 - x^2}{x+y}$; the domain of f is $\{(x,y) : x \neq -y\}$ since f is a rational function.

So, $f(x,y)$ is undefined on the line $y = -x$.

This means that $f(x,y)$ is not defined on any disk of radius $\delta > 0$ around $(0,0)$; thus there is no such $\delta > 0$ where $|f(x,y) - \#| <$ a finite number.

So, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2-x^2}{x+y}$ D.N.E. (does not exist).

4. (7pts.) Is the following function continuous at $(2,3)$? **Justify your answer.**

$$f(x,y) = \begin{cases} 0 & \text{if } (x,y) = (0,0) \\ 2 & \text{if } (x,y) = (2,3) \\ \frac{3xy}{x^2+y^2} & \text{if } (x,y) \notin \{(0,0),(2,3)\} \end{cases}$$

$$\lim_{(x,y)\to(2,3)} f(x,y) = \lim_{(x,y)\to(2,3)} \frac{3xy}{x^2+y^2} = \frac{3(2)(3)}{2^2+3^2}$$ bec $\dfrac{3xy}{x^2+y^2}$ is cont. at $(2,3)$.

$$= \frac{18}{13}$$

And $f(2,3) = 2$.

Since $f(2,3) \neq \displaystyle\lim_{(x,y)\to(2,3)} f(x,y)$, f is NOT continuous at $(2,3)$