

5. (15pts.) Let

$$f(x, y) = \frac{1}{\sqrt{16y^2 - 25x^2}}$$

(a) Find the domain of $z = f(x, y)$; justify your answer **without** any graphing softwares or tools.

need $16y^2 - 25x^2 > 0 \Rightarrow (4y - 5x)(4y + 5x) > 0$
 $\Rightarrow (4y - 5x > 0 \text{ and } 4y + 5x > 0) \text{ or } (4y - 5x < 0 \text{ and } 4y + 5x < 0)$
 $\Rightarrow \underline{(y > \frac{5}{4}x \text{ and } y > -\frac{5}{4}x)}$ or $\underline{(y < \frac{5}{4}x \text{ and } y < -\frac{5}{4}x)}$

So, the domain, $D = \{(x, y) : 16y^2 - 25x^2 > 0\}$
 $= \{(x, y) : (y > \frac{5}{4}x \text{ and } y > -\frac{5}{4}x) \text{ or } (y < \frac{5}{4}x \text{ and } y < -\frac{5}{4}x)\}$

(b) Find the range of $z = f(x, y)$; justify your answer **without** any graphing tools.

$z = \frac{1}{\sqrt{\quad}} > 0$ So, the range is $\{z : z > 0\}$.

(c) Sketch the domain on the xy plane given in **Figure 1**, **without** any graphing tools but based on your answer in part (a).

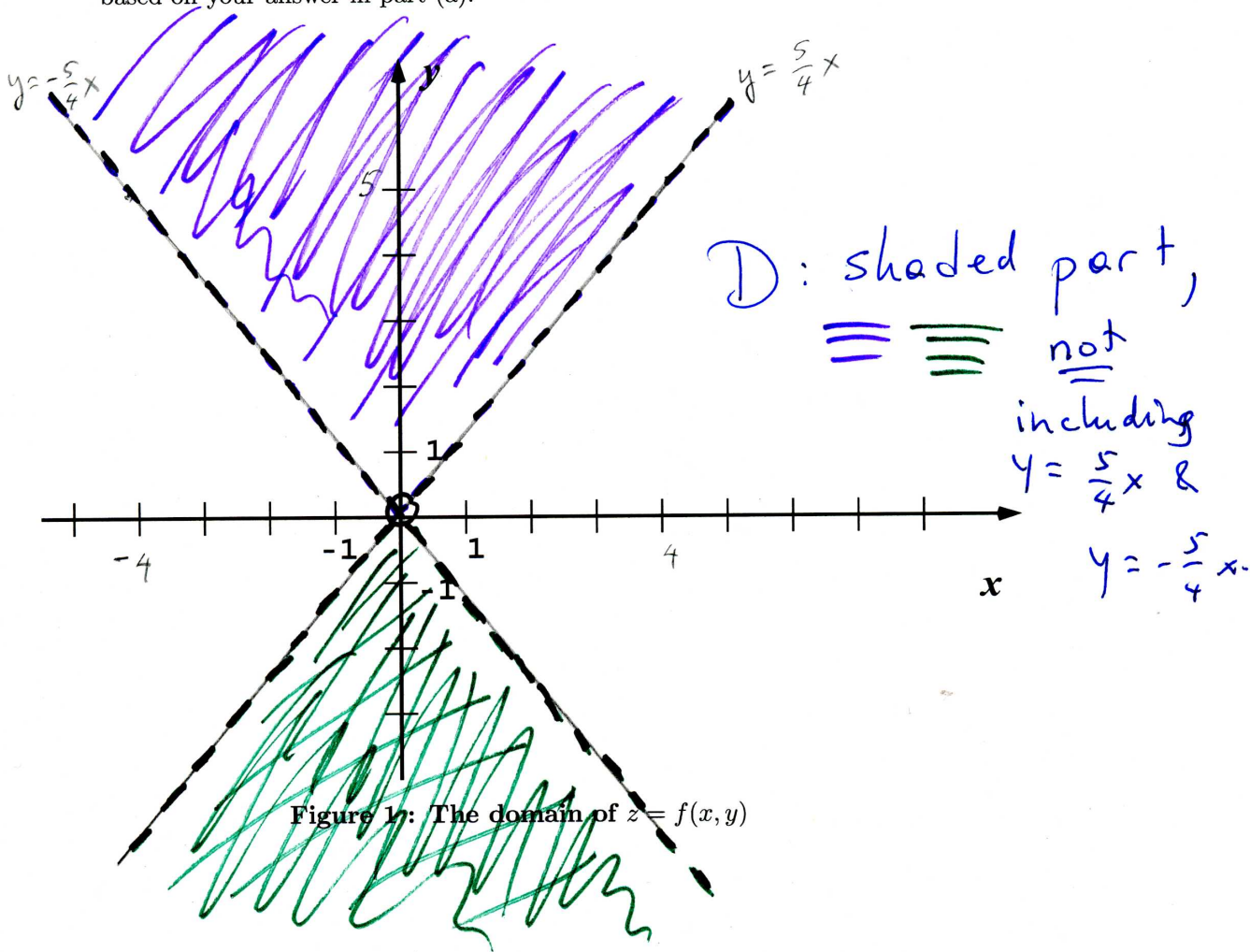


Figure 1: The domain of $z = f(x, y)$