6. (7pts.) (No mathematica; no calculator.) Show your work!

On the axes provided in **Figure 2**, sketch and **identify** several level curves of the function:

$$f(x,y) = \begin{cases} \sqrt{x^2+y^2} & \text{if } x \geq 0 \\ |y| & \text{if } x < 0 \end{cases}$$

Level curves are: $f(x,y) = k$, $k \in \mathbb{R}$

① $k=0$, (i) if $x<0$ then $|y|=0$
(ii) if $x \geq 0$ " $\sqrt{x^2+y^2}=0$
$\Rightarrow (x,y) = (0,0)$

② $k<0$, (i) if $x<0$ then $|y| = -|k|$, no curves
(ii) if $x \geq 0$, $\sqrt{x^2+y^2} = -|k|$, no curves

③ $k>0$,
(i) If $x<0$, $|y| = |k|$
$\Rightarrow y = \pm k$, horizontal line
(ii) If $x \geq 0$, $\sqrt{x^2+y^2} = |k|$
$\Rightarrow x^2+y^2 = |k|^2$, circles of rad k.

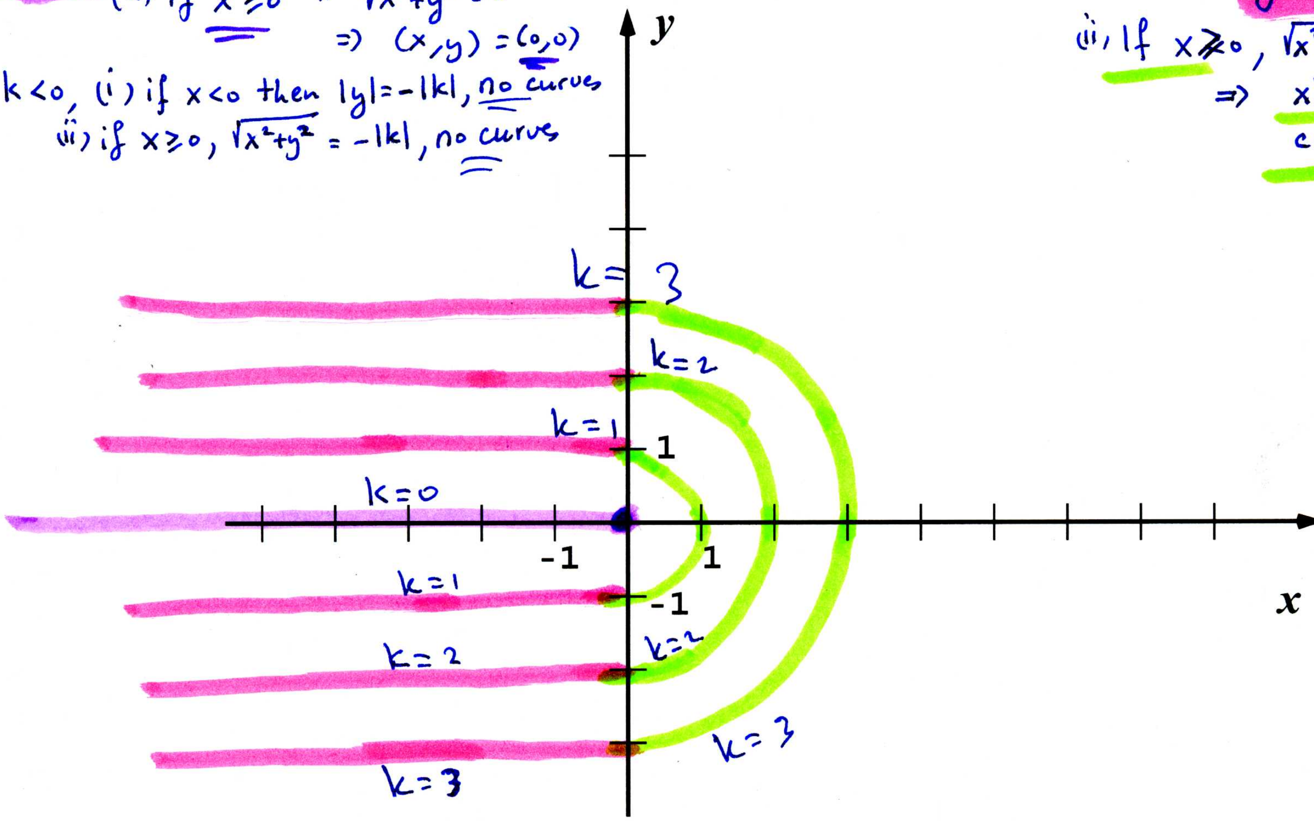

Figure 2 : Level curves of $z = f(x,y)$

7. (7pts.) Use differentials to approximate the value of:

$$\sqrt{9.02}(\sqrt[3]{7.95}+1)$$

(Hint: Define a function $z = f(x,y)$ so that the above problem is equivalent to using differentials to approximate $f(x+\Delta x, y+\Delta y)$)

Let $f(x,y) = \sqrt{x}\,(\sqrt[3]{y}+1)$. Then $f_x = \dfrac{\sqrt[3]{y}+1}{2\sqrt{x}}$ & $f_y = \dfrac{\sqrt{x}}{3y^{2/3}}$

Since $f(x+\Delta x, y+\Delta y) \approx f(x,y) + f_x(x,y)\Delta x + f_y(x,y)\Delta y$, for $\Delta x, \Delta y$ small,

$\sqrt{9.02}\,(\sqrt[3]{7.95}+1) = f(9.02, 7.95) \approx f(9,8) + f_x(9,8)\Delta x + f_y(9,8)\Delta y$ where $\Delta x = .02$, $\Delta y = -.05$

$$= \sqrt{9}(\sqrt[3]{8}+1) + \frac{\sqrt[3]{8}+1}{2\sqrt{9}}(.02) + \frac{\sqrt{9}}{3(8)^{2/3}}(-.05)$$

$$= 3(2+1) + \frac{2+1}{2(3)}(.02) + \frac{3}{3(4)}(-.05)$$

$$= 9 + .01 - .0125 = 8.9975$$