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8. (26pts.) Let

$$z = f(x, y) = e^{-y}(x^2 - y^2)$$

(a) (3pts.) Find $\frac{\partial z}{\partial x} =$ $2xe^{-y}$

(b) (3pts.) Find $\frac{\partial z}{\partial y} =$ $-e^{-y}(x^2-y^2) - 2ye^{-y} = -e^{-y}(x^2-y^2+2y)$

(c) (3pts.) Find $\frac{\partial}{\partial x}(\frac{\partial z}{\partial x}) =$ $2e^{-y} = f_{xx}(x,y)$

(d) (3pts.) Find $\frac{\partial}{\partial y}(\frac{\partial z}{\partial x}) =$ $-2xe^{-y} = f_{xy}(x,y)$

$f_{yy}(x,y)$ $\Big\{$ (e) (3pts.) Find $\frac{\partial}{\partial y}(\frac{\partial z}{\partial y}) =$ $e^{-y}(x^2-y^2+2y) - e^{-y}(-2y+2)$

$= e^{-y}[x^2-y^2+2y+2y-2] = e^{-y}[x^2-y^2+4y-2]$

(f) (3pts.) Why should your answer in (d) be the same as $\frac{\partial}{\partial x}(\frac{\partial z}{\partial y}) =$?

Bec $\frac{\partial^2 z}{\partial x \partial x}$ & $\frac{\partial^2 z}{\partial y \partial x}$ are continuous functions & by Clairaut's Thm, they are the same.

(g) (8pts.) Find all local minimum points, local maximum points, and saddle points of f. **Clearly identify which is which.**

cpts: $\left\{\begin{array}{l} f_x = 0 \\ f_y = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 2xe^{-y} = 0 \\ -e^{-y}(x^2-y^2+2y) = 0 \end{array}\right\} \Rightarrow$ $x = 0$ & $-y^2+2y = 0 \Rightarrow y(-y+2) = 0$

$\Rightarrow y = 0$ or $y = 2$

So, cpts are $(0,0)$ & $(0,2)$.

$D(0,0) = f_{xx}(0,0)\, f_{yy}(0,0) - [f_{xy}(0,0)]^2 = (2)(-2) - (0)^2 < 0$

So, $\boxed{(0,0) \text{ yields a saddle point of } f.}$

$D(0,2) = f_{xx}(0,2) f_{yy}(0,2) - [f_{xy}(0,2)]^2 = (2e^{-2})e^{-2}(-4+8-2) - 0^2 > 0.$

& $f_{xx}(0,2) = 2e^{-2} > 0$.

So, $\boxed{(0,2) \text{ yields a local minimum point of } f.}$