CSci 1302 Assignment 5

Due Wednesday, Oct. 12

Problem 1 (5 points). Which of the following formulas are equivalent to $\tilde{}\,\forall x.\exists y.p(x,y)$? Please explain your reasoning for each formula below.

1. $\forall x.\exists y.\tilde{}\,p(x,y)$
2. $\tilde{}\,\exists y.\forall x.p(x,y)$
3. $\exists x.\tilde{}\,\exists y.p(x,y)$
4. $\exists x.\forall y.p(x,y)$
5. $\exists x.\forall y.\tilde{}\,p(x,y)$

Problem 2 (24 points: 1,2,3 are 2 points each, the rest are 3 points each). Assume the following:

1. A chess team A consists of Adam, Alice, and Ann. A(x) means that the person x is on the team A.
2. A chess team B consists of Bob and Beth. B(x) means that the person x is on the team B.
3. The relation wonAgainst(x,y) means that x has won against y at least once. Some people never played against each other, so no comparison is given for such pairs. The following are true statements:
   (a) Adam has won against Ann and Alice.
   (b) Alice has won against Ann.
   (c) Beth has won against Adam and Bob.
   (d) Bob has won against Alice.
   (e) Ann has won against Alice.

Based on the information above, are the following true or false statements? Prove your answers by using tables. You may show only a part of the table if it is sufficient to prove or disprove the statement.

1. $\exists x.A(x) \to wonAgainst(x, Beth)$
2. $\forall x.wonAgainst(Adam, x) \lor wonAgainst(Beth, x)$
3. $\forall x.B(x) \lor \sim wonAgainst(x, Beth)$
4. $\forall x.\exists y.A(x) \to wonAgainst(y, x)$
5. $\forall x.\exists y.A(x) \land wonAgainst(y, x)$
6. $\exists x.\forall y.wonAgainst(x, y) \lor wonAgainst(y, x)$
7. $\exists x.\exists y.wonAgainst(x, y) \land wonAgainst(Adam, x)$
8. $\forall x.\forall y.A(x) \lor B(y)$
9. $\forall x.\exists y.\sim wonAgainst(x, y)$

Problem 3 (Extra credit, 5 points) Use the system in Problem 2 (without adding any new relations) and give an example of $p(x, y)$ such that $\forall x.\exists y.p(x, y)$ is true, but $\exists y.\forall x.p(x, y)$ is not.

Problem 4 (12 points). Prove the following arguments. The domain for all problems is $\mathbb{Z}$ - the set of all integers.

1. $\forall x.\exists y.x^2 = y$

$\therefore \exists z.5^2 = z$

1. $\forall x.(x \neq 1 \land x \neq 0) \to x^2 > x$
2. $\exists y.y \neq 1 \land y \neq 0$

$\therefore \exists z.z^2 > z$

Hint: when introducing the existential quanifier, replace only one occurrence of the constant by a variable, but not the other:

1. $\forall z.isDivisible(z, 1) \land isDivisible(z, z)$

$\therefore \exists y.isDivisible(y, 33)$

You may use the fact that $x < y \equiv y > x$:

1. $\forall x.((x > 0) \lor (x < 0)) \leftrightarrow x^2 > 0$
2. $\sim (0 < 0)$
3. $\forall x.(x < 0) \lor (x > 0) \lor (x = 0)$

$\therefore x^2 = 0$