CSci 1302 Assignment 3

Due Wedn., February 11th, 2004

Note: the symbol <≡> stands for logical equivalence.

Problem 1 (5 points). Given the Negation law, both De Morgan's laws, and the Distributivity law $p \vee (q \wedge r) <\equiv> (p \vee q) \wedge (p \vee r)$, prove the second Distributivity law:

$$p \wedge (q \vee r) <\equiv> (p \wedge q) \vee (p \wedge r).$$

Use a transformational proof, not truth tables.

Hint: use the same approach as in the proofs in Example 4.3.

Problem 2 (6 points). Exercise 4.4 p. 54.

Problem 3 (20 points). Prove the following:

1. $(p \Rightarrow q) \vee (p \Rightarrow r) \vee p$ is a tautology,
2. $(p \Rightarrow q) \wedge (p \Rightarrow r) \wedge p <\equiv> p \wedge q \wedge r$,
3. $(p \Rightarrow q) \wedge \neg q <\equiv> \neg(p \vee q)$,
4. $p \vee (q \wedge r \wedge s) <\equiv> (p \vee q) \wedge (p \vee r) \wedge (p \vee s)$,
5. $(p \wedge q \wedge r) \vee s <\equiv> (p \vee s) \wedge (q \vee s) \wedge (r \vee s)$.

Try to make your proof short. Proofs that are too long (i.e. have unnecessary steps) may get lower grades.

Problem 4 (14 points total). Consider a connective "exclusive OR", denoted by $\oplus$.

Question 1 (5 points). Using the symbol $\oplus$, write down the following properties:

1. idempotence of $\oplus$
2. commutativity of $\oplus$
3. associativity of $\oplus$
4. distributivity of $\oplus$ over $\wedge$
5. distributivity of $\Rightarrow$ over $\oplus$

You don't need to worry about whether these properties are true or false. You don't even need to know (yet) what $\oplus$ stands for.

Question 2 (4 points). Recall that $\oplus$ is defined as follows: $p \oplus q$ if and only if $(p \vee q) \wedge \neg(p \wedge q)$. Rewrite the properties 1, 2, and 3 above without using the notation $\oplus$.

Extra credit, 4 points. Rewrite properties 4 and 5 without using the notation $\oplus$.

Question 3 (5 points). For the first two properties in Question 2 (idempotence and commutativity of $\oplus$) either give a transformational proof, or show that the property doesn't hold.