CSci 1302 Assignment 11

Due Friday, May 5

Problem 1 (6 points). You are given three sets, $A = \{a, b, c\}$, $B = \{1, 2, 3\}$, and $C = \{red, white, blue\}$, and the following relations:

- $R \subseteq A \times B = \{(a, 1), (b, 1), (c, 3)\}$,
- $S \subseteq B \times C = \{(2, red), (2, blue), (3, red)\}$,
- $T \subseteq C \times A = \{(red, a), (white, c)\}$.

For each of the following operations compute the result if the operation makes sense, or, if it doesn't make sense, please explain why.

1. $R; S$
2. $S; T$
3. $R; R^{-1}$
4. $R^{-1}; R$
5. $R^{-1}; T^{-1}$
6. $R; T$

Problem 2 (4 points). Exercises 4, 11 p. 608.

Problem 3 (6 points). For each relation on natural numbers $\mathbb{N} = \{1, 2, 3, \dots\}$ defined below please answer the following questions:

- Is the relation reflexive?
- Is the relation symmetric?
- Is the relation antisymmetric?
- Is the relation transitive?

Please explain the negative answers briefly.

1. $R = \{(n, m) \mid n + m \text{ is even}\}$
2. $R = \{(n, m) \mid n + m \text{ is odd}\}$
3. $R = \{(n, m) \mid n \text{ is even}, m \text{ is even}\}$

Problem 4 (5 points) You are given relation $R = \{(a, b), (b, c), (c, b), (d, c)\}$ on the universal set $U = \{a, b, c, d, e\}$. Please construct the following:

- the reflexive closure of R.
- the symmetric closure of R.
- the transitive closure of R.
- the "equivalence closure" of R (i.e. the smallest equivalence relation that contains R).

You may list pairs included in the resulting relations or draw them (each on a separate diagram).

Problem 5 (2 points). Is symmetric closure of a transitive relation transitive? If yes, please prove it. If not, please give a counterexample.

Problem 6 (3 points). Exercises 6, 7, 9 p. 647.

Problem 7 (2 points). Exercises 17, 18 p. 663.

Problem 8 (6 points). Exercise 2 p. 680.

Problem 9 (2 points). Exercises 3b, 5b p. 696.

Problem 10 (6 points). Exercise 19 p. 696.

That's all!