CSci 4651 Fall 2008
Problem Set 3: Context-Free Grammars and λ-calculus
Due Wedn., Sept. 24 in class.

Problem 1. Consider the following grammar for boolean expressions:

$$\begin{array}{lcl} e & \rightarrow & b \mid v \mid e \wedge e \mid e \vee e \mid \neg e \\ b & \rightarrow & \text{true} \mid \text{false} \\ v & \rightarrow & x \mid y \mid z \end{array}$$

Question 1 (4 points). Without making any assumptions about the precedence of the operations, please answer the following about each of the 4 expressions below:

- Does this expression belong to the language defined by the grammar?
- If yes, please draw a parse tree for it in the grammar.
- If it doesn't belong, briefly explain why.

The expressions are as follows:

1. $x \wedge \text{false}$
2. $x \neg y \wedge z$
3. $x \vee x \wedge y$
4. $x \vee \neg \text{true}$

Question 2 (2 points). Show that the grammar is ambiguous by giving an example of an expression that has two different parse trees; draw the parse trees.

Question 3 (12 points). Write the corresponding unambiguous grammar that would enforce the left associativity of $\vee$ and $\wedge$ and the following precedence:

- $\neg$ has the highest precedence
- $\wedge$ has the second-highest precedence
- $\vee$ has the lowest precedence

You don't need to include parentheses in your grammar. Test your grammar on the following expressions, draw the parse tree for each one, check that it is unique and correct:

- $\neg x \vee y$
- $x \vee y \wedge z$
- $x \wedge \neg y \wedge z$

Problem 2 (4 points). Which of the following terms are α-equivalent to $\lambda x.\lambda y.xyz$?

1. $\lambda y.\lambda x.yxz$
2. $\lambda x.\lambda x.xxz$
3. $\lambda z.\lambda y.zxz$
4. $\lambda y.\lambda z.yzx$

Please explain your answer.