Questions

1. Sketch $f(x) = -2x^2 + 4x + 1$.

2. Sketch $f(x) = -x^2 + 2x - 6$.

3. Solve $x^2 - 4x + 1 < 0$.

4. Solve $x + 6 > 5x^2$.

5. $f(x) = x^3 - 3x - 10$.

(a) Solve $f(x) = 0$.
(b) Solve $f(x) = -10$.
(c) Solve $f(x) > 0$.
(d) Solve $f(x) \leq 0$.
(e) Write f in the form $f(x) = a(x-h)^2 + k$ and describe the graph of f as a transformation of $y = x^2$.
(f) Graph f and state the domain, range, and the maximum or minimum y-coordinate on the graph.
(g) What is the relationship between the graph of f and the answers to parts (c) and (d)?
(h) Find the intercepts, axis of symmetry, vertex, opening, and intervals on which f is increasing or decreasing.

6. If a football is kicked straight up with an initial velocity of 128 ft/sec from a height of 5ft, then the height above the Earth is a function of time given by $h(t) = -16t^2 + 128t + 5$. What is the maximum height reached by the football?

7. If an archer shoots an arrow straight up in the air with an initial velocity of 160 ft/sec from a height of 8 ft, then the arrow's height above the ground is given by the function $h(t) = -16t^2 + 160t + 8$. What is the maximum height reached by the arrow? How long does it take for the arrow to reach the ground?

① $f(x) = -2x^2+4x+1.$

Zeros: $-2x^2+4x+1=0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{16-4(-2)(1)}}{2(-2)}$$
$$= \frac{-4 \pm \sqrt{24}}{-4}$$
$$= 1 \pm \frac{\sqrt{6}}{2}$$

(Note $1-\frac{\sqrt{6}}{2} = -0.224745 < 0$)

Note: This can be replaced by saying $a=-2<0$ → opens down.

End behaviour: If $|x|$ is large, $f(x) \sim -2x^2$. Therefore

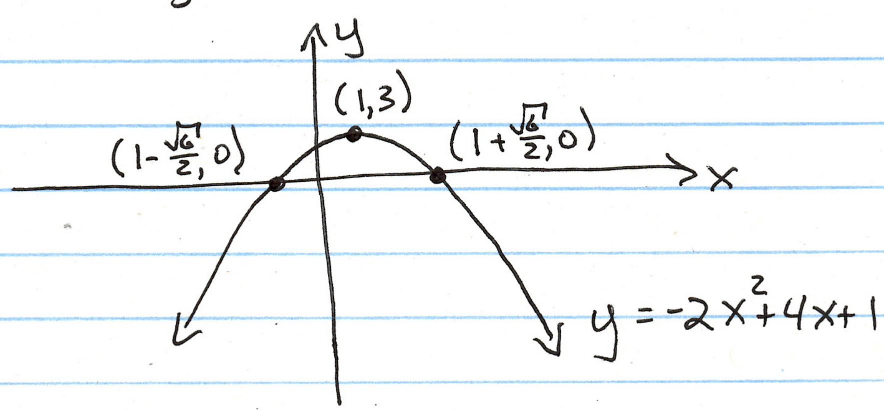
$$\lim_{x\to\infty} f(x) = -\infty$$

$$\lim_{x\to-\infty} f(x) = -\infty.$$

Vertex: is halfway between x-intercepts.

$$x = \cancel{\frac{1+\sqrt{6}}{2}}\ \frac{\left(1+\frac{\sqrt{6}}{2}\right)+\left(1-\frac{\sqrt{6}}{2}\right)}{2} = 1$$

$$y = f(1) = -2(1)^2+4(1)+1 = 3$$



(2) $f(x) = -x^2+2x-6$

sketch by completing the square to get vertex form,
$f(x) = a(x-h)^2+k$ Vertex (h,k)

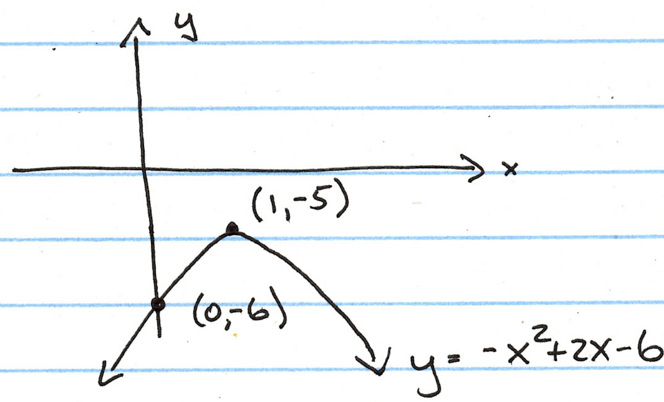
$$f(x) = -(\underbrace{x^2-2x+1}-1)-6$$
$$= -((x-1)^2-1)-6$$
$$= -(x-1)^2-5 \longrightarrow \text{Vertex } (1,-5)$$

Since $a=-1<0$, quadratic opens down.

<u>Zeros:</u> $-(x-1)^2-5=0$

$$(x-1)^2=-5$$
$$x-1=\pm\sqrt{-5} \quad \text{no real roots.}$$

no x-intercepts.



(3) $f(x) = x^2 - 4x + 1 < 0.$

<u>zeros:</u> $x^2 - 4x + 1 = 0$

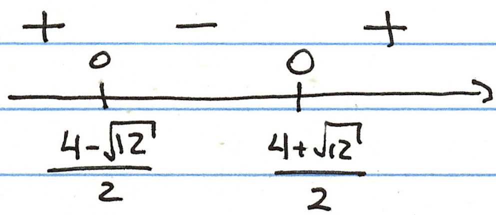
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm 16 - 4(1)(1)}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2}$$

Since $a = 1 > 0$, $f(x)$ opens up. Therefore $\lim\limits_{x \to \infty} f(x) = \infty$

$\lim\limits_{x \to -\infty} f(x) = +\infty.$



from signchart, $f(x) < 0$ for $x \in \left(\frac{4 - \sqrt{12}}{2}, \frac{4 + \sqrt{12}}{2}\right)$

which simplifies to $x \in (2 - \sqrt{3}, 2 + \sqrt{3})$.

(4) ~~f(x)~~ $x + 6 > 5x^2$

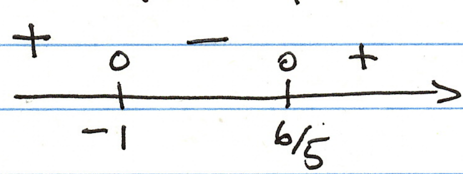
$f(x) = 5x^2 - x - 6 < 0$

<u>zeros:</u> $f(x) = (x+1)(5x-6) = 0$ (factor)

$x = -1$ or $x = 6/5$.

$f(x)$ opens up since $a = 1 > 0$. Therefore $\lim\limits_{x \to \infty} f(x) = \infty$

$\lim\limits_{x \to -\infty} f(x) = \infty.$



So $f(x) < 0$ when $x \in (-1, 6/5)$.

5) $\boxed{f(x) = x^2 - 3x - 10.}$

a) $f(x) = x^2 - 3x - 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$$
$$= \frac{3 \pm 7}{2} = 5 \text{ or } -2.$$

b) $f(x) = x^2 - 3x - 10 = -10$

$$x^2 - 3x = 0$$
$$x(x-3) = 0$$
$$x = 0 \text{ or } x = 3.$$

c) $f(x) = x^2 - 3x - 10 > 0$

sign chart: from a),

$$f(x) = (x-5)(x+2) > 0$$

f(x) changes sign at zeros, and opens up since $a = 1 > 0$.

$$\begin{array}{ccccc} + & 0 & - & 0 & + \\ & -2 & & 5 & x \end{array}$$

$f(x) > 0$ for $x \in (-\infty, -2) \cup (5, \infty)$.

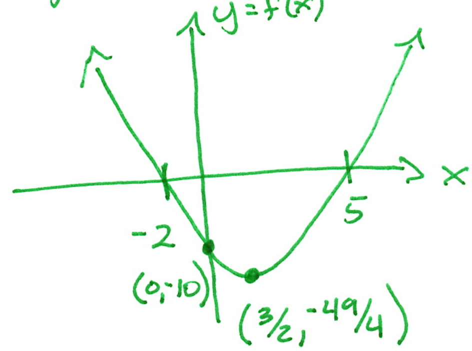
d) $f(x) \leq 0$ for $x \in [-2, 5]$ (using sign chart in c)).

e) $f(x) = x^2 - 3x - 10$

$$= \underbrace{x^2 - 3x + \frac{9}{4}} - \frac{9}{4} - 10$$
$$= \left(x - \frac{3}{2}\right)^2 - \frac{49}{4}$$

$f(x)$ is $y = x^2$ shifted right $3/2$ and down $49/4$.

f)



Domain: $x \in \mathbb{R}$

Range: $y \in \left[-\frac{49}{4}, \infty\right)$

g) I used the zeros and End behaviour to get the sign chart, which was what was used to answer c) and d).

h) x intercepts: $x = -2, 5$

y intercept: $y = -10$.

axis of symmetry: $x = 3/2$

vertex: $(3/2, -49/4)$

opens up

decreasing: $x \in (-\infty, 3/2)$

increasing: $x \in (3/2, \infty)$.

(6) $h(t) = -16t^2 + 128t + 5$.

h: height

t: time.

sketch: $a = -16 < 0$, so this is a parabola opening down.

Complete square to get vertex form.

$$h(t) = -16\left[\underbrace{t^2 - 8t + 16}-16\right] + 5$$

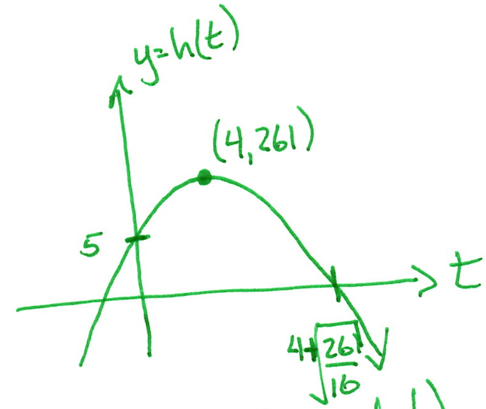
$$= -16\left[(t-4)^2 - 16\right] + 5$$

$$= -16(t-4)^2 + 256 + 5$$

$$= -16(t-4)^2 + 261$$

Compare to $y = a(t-h)^2 + k$ Note this h is not the same as h(t).

$\rightarrow$ Vertex $= (h, k) = (4, 261)$



(obviously not to scale!).

Zeros: $-16(t-4)^2 + 261 = 0$

$$(t-4)^2 = \frac{261}{16}$$

$$t = 4 \pm \sqrt{\frac{261}{16}}.$$

Note: the zeros aren't needed to answer this question.

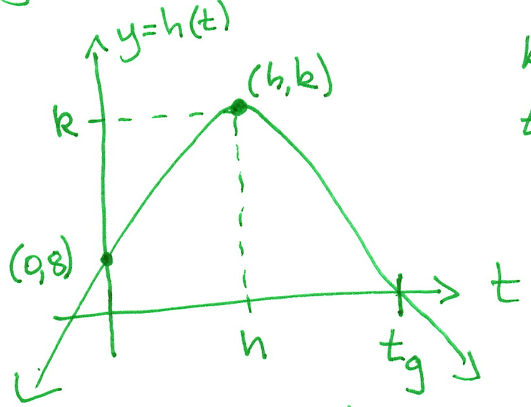
After 4 seconds, the ball achieves max height of 261 ft.

(7) $h(t) = -16t^2 + 160t + 8$.

$a = -16 < 0$ so $h(t)$ opens down.

$h(0) = 8$.

This is enough information to get a rough sketch!



k max height.

t_g time when arrow hits ground.

Complete square to get (h,k)

$$h(t) = -16[\underbrace{t^2 - 10t + 25}, -25] + 8$$

$$= -16[(t-5)^2 - 25] + 8$$

$$= -16(t-5)^2 + 408.$$

Compare to $y = a(t-h)^2 + k$ $\rightarrow$ $h = 5$, $k = 408$.

note: this h is not the same h as $h(t)$.

so arrow reaches max height of $k = 408$ ft at $h = 5$ seconds.

It hits ground when $h(t) = 0 = -16t^2 + 160t + 8$.

$$\rightarrow -16(t-5)^2 + 408 = 0$$

$$(t-5)^2 = \frac{51}{2}$$

$$t - 5 = \pm\sqrt{\frac{51}{2}}$$

$$t = 5 \pm \sqrt{\frac{51}{2}}$$

So $t_g = 5 + \sqrt{\frac{51}{2}}$ seconds is when it hits ground

$= 10.0498$ seconds

(it makes sense to convert to a decimal here).