Questions

1. Use the Rational Zero Theorem to find all possible rational zeros for the polynomial

$$P(x) = 4x^3 - 10x^2 + 4x + 5$$

2. Find all the real and imaginary zeros for the polynomial

$$P(x) = 4x^3 - 10x^2 + 4x + 5$$

3. Find all the real and imaginary zeros for the polynomial

$$P(x) = 2x^4 + 5x^3 + 3x^2 + 15x - 9$$

4. Joan intends to make an 18 in^3 open top box out of a 6in by 7in piece of copper by cutting equal squares (x in by x in) out of each corner and folding up the sides. Write the difference between the intended volume and the actual volume as a function of x. What value should x be to get a box with volume 18in^3?

(1) $P(x) = 4x^3 - 10x^2 + 4x + 5$

factors of 5: $\pm 1, \pm 5$

factors of 4: $\pm 1, \pm 2, \pm 4$

Take ratio

Potential Rational zeros: $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{4}, \pm \frac{5}{4}$.

Note: everything always has a $\pm$ on it, so if you prefer to write

factors of 5: $\pm(1, 5)$

factors of 4: $\pm(1, 2, 4)$

Potential Rational Zeros: $\pm\left(1, 5, \frac{1}{2}, \frac{5}{2}, \frac{1}{4}, \frac{5}{4}\right)$

that would be OK!

(2) Continuing, we look for $f(c) = 0$ where c comes from above list.

$P(1) = 4(1)^3 - 10(1)^2 + 4(1) + 5 = 3$ so $x - 1$ not factor

$P(-1) = 4(-1)^3 - 10(-1)^2 + 4(-1) + 5 = -13$ so $x + 1$ not factor

$P(1/2) = 4(\frac{1}{2})^3 - 10(\frac{1}{2})^2 + 4(\frac{1}{2}) + 5 = 5$ so $x - 1/2$ not factor

$P(-1/2) = 4(-\frac{1}{2})^3 - 10(-\frac{1}{2})^2 + 4(-\frac{1}{2}) + 5 = 0$ so $x + 1/2$ is a factor!

If $x + 1/2$ is a factor, $2x + 1$ is also a factor. I prefer to avoid working with fractions so I make this change here.

Now, divide out the factor

$$\begin{array}{r l}
 & 2x^2-6x+5 \\
2x+1 \,\big)\! & \overline{4x^3-10x^2+4x+5} \\
 & \underline{4x^3+2x^2} \qquad \text{subtract} \\
 & -12x^2+4x \\
 & \underline{-12x^2-6x} \qquad \text{subtract} \\
 & 10x+5 \\
 & \underline{10x+5} \qquad \text{subtract} \\
 & 0 \qquad \text{(remainder is zero)}
\end{array}$$

Therefore, $4x^3-10x^2+4x+5 = (2x+1)(2x^2-6x+5)$.

$P(x)$ has zeros $\quad 2x+1=0 \qquad 2x^2-6x+5=0$

$$x = -\tfrac{1}{2} \qquad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{36-4(2)(5)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{-4}}{4}$$

$$= \frac{6 \pm 2i}{4}$$

$$= \frac{3 \pm i}{2}$$

All zeros of $P(x)$ are $x = -\tfrac{1}{2}, \frac{3-i}{2}, \frac{3+i}{2}$.

③ $P(x) = 2x^4 + 5x^3 + 3x^2 + 15x - 9$

Factors of -9 : $\pm(1, 3, 9)$

Factors of 2 : $\pm(1, 2)$

Potential Rational zeros: $\pm\left(1, 3, 9, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}\right)$.

$P(1) = 2(1)^4 + 5(1)^3 + 3(1)^2 + 15(1) - 9 = 16 \neq 0$. So $x-1$ is not factor.

$P(\frac{1}{2}) = 2(\frac{1}{2})^4 + 5(\frac{1}{2}) + 3(\frac{1}{2}) + 15(\frac{1}{2}) - 9 = 0$

So $x - \frac{1}{2}$ is factor.

$\rightarrow$ $2x - 1$ is a factor.

$$
\begin{array}{rl}
 & x^3 + 3x^2 + 3x + 9 \\
2x-1 \,\big) & 2x^4 + 5x^3 + 3x^2 + 15x - 9 \\
 & \underline{2x^4 - x^3} \qquad \text{subtract} \\
 & 6x^3 + 3x^2 \\
 & \underline{6x^3 - 3x^2} \qquad \text{subtract} \\
 & 6x^2 + 15x \\
 & \underline{6x^2 - 3x} \qquad \text{subtract} \\
 & 18x - 9 \\
 & \underline{18x - 9} \qquad \text{subtract} \\
 & 0 \qquad \text{(remainder is zero).}
\end{array}
$$

Therefore, $P(x) = (2x-1)(x^3 + 3x^2 + 3x + 9)$.

Now, factor $f(x) = x^3 + 3x^2 + 3x + 9$!

$f(x) = x^3+3x^2+3x+9$

Factors of 9: $\pm(1,3,9)$

Factors of 1: ± 1

Potential Rational ~~Factors~~ Zeros!: $\pm(1,3,9)$

$f(-3) = (-3)^3+3(-3)^2+3(-3)+9=0$ so $x+3$ *is* a factor!

$$
\begin{array}{r|l}
 & x^2 + 3 \\
\hline
x+3 & x^3+3x^2+3x+9 \\
 & \underline{x^3+3x^2} \\
 & \qquad\quad 3x+9 \\
 & \qquad\quad \underline{3x+9} \\
 & \qquad\qquad 0
\end{array}
$$

subtract

subtract

(remainder is zero).

Therefore, $f(x) = (x+3)(x^2+3)$

$P(x) = (2x-1)(x+3)(x^2+3) = 0$

$2x-1=0$ or $x+3=0$ or $x^2+3=0$

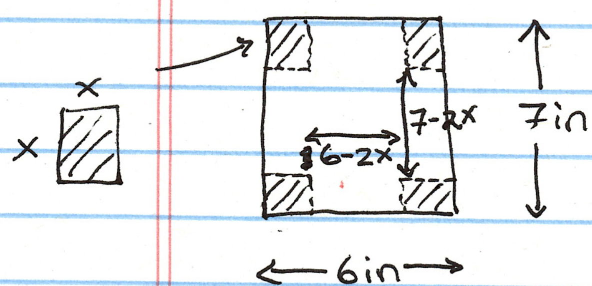
$x = \frac{1}{2}$ $\qquad x=-3$ $\qquad x^2=-3$

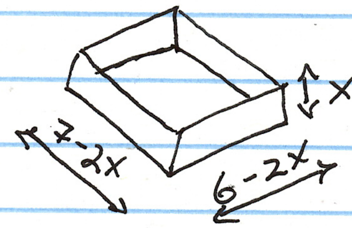
$x = \pm\sqrt{-3}$

$= \pm\sqrt{3}\,i$

All zeros of $P(x)$ are $x = \frac{1}{2}, -3, \pm\sqrt{3}\,i$.

④ Here is a sketch of how the box will be made:





~~Domain: x>0~~
~~6-2x>0~~
~~7-2x>0~~

Made a mess of it!

Volume of box = lwh

$= (7-2x)(6-2x)x$

Domain:

$x>0$	$x>0$
$6-2x>0$	$x<3$
$7-2x>0$	$x<7/2$

$\rightarrow x \in (0,3)$

$P(x)$ = intended volume − actual volume

$= 18 - (7-2x)(6-2x)x$

$= 18 - (42x - 26x^2 + 4x^3)$

$= -4x^3 + 26x^2 - 42x + 18.$

To get volume of 18, we need to solve $P(x) = 0$.

factors of 18: $\pm(1, 2, 3, 6, 9, 18)$

factors of -4: $\pm(1, 2, 4)$

Potential Rational zeros: $\pm\left(1, 2, 3, 6, 9, 18, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{1}{4}, \frac{3}{4}, \frac{9}{4}\right)$

(note I eliminated duplicates).

$P(3/2) = 0$ so $x - 3/2$ is a factor

$\longrightarrow 2x-3$ is a factor.

$$
\begin{array}{r l}
 & -2x^2 + 10x - 6 \\
2x-3 \,\big) & -4x^3 + 26x^2 - 42x + 18 \\
 & \underline{-4x^3 + 6x^2} \qquad \text{subtract} \\
 & 20x^2 - 42x \\
 & \underline{20x^2 - 30x} \qquad \text{subtract} \\
 & -12x + 18 \\
 & \underline{-12x + 18} \qquad \text{subtract} \\
 & 0 \qquad \text{(remainder 0)}
\end{array}
$$

Therefore, $P(x) = (2x-3)(-2x^2+10x-6)$

$= -2(2x-3)(x^2-5x+3)$

$P(x)=0$ when $2x-3=0$ or $x^2-5x+3=0$

$x = 3/2$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25-4(1)(3)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{13}}{2}$$

Note: $x = \frac{5-\sqrt{13}}{2} \sim 0.697224$

$x = \frac{5+\sqrt{13}}{2} \sim 4.30278.$ ← This won't work since x cannot be bigger than $\frac{6}{2}=3$!

So there are ~~three~~ two possible values of x to create a box with volume 18 in^3:

$$x = \frac{3}{2},\ \cancel{\frac{5+\sqrt{13}}{2}},\ \text{or}\ \frac{5-\sqrt{13}}{2}.$$

↑ not physically possible.