Questions

Find all real and imaginary solutions for each equation.

1. $w^4 + 8w = 0.$

2. $x - \dfrac{\sqrt{1-3x}}{2} = 0.$

3. $\dfrac{1}{x} - \dfrac{2}{\sqrt{2x+1}} = 0.$

4. $\sqrt{2x+5} + \sqrt{x+6} = 9.$

5. $(s-1)^{-1/2} = 2.$

6. $\left(\dfrac{2c-3}{5}\right)^2 + 2\left(\dfrac{2c-3}{5}\right) = 8.$

7. $\dfrac{1}{(5x-1)^2} + \dfrac{1}{5x-1} = 12.$

8. $|x^2 + 5x| = |3 - x^2|.$

9. Will, Nanc, and Ed met at the lodge and went hiking. Will began hiking west at 4mph at 8am. At 10am, Nanc began hiking north at 5mph. Ed went east at 12mph, also leaving at 10am. At what time was the distance between Nanc and Ed 14miles greater than the distance between Nanc and Will? Just try to set up the equation that would need to be solved. It is a wacky looking equation!

① $w^4 + 8w = 0$

$w(w^3 + 8) = 0$

$w(w^3 + 2^3) = 0$ use sum of cubes formula

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a = w$ $\quad w^3 + 2^3 = (w+2)(w^2 - 2w + 4)$

$b = 2$

$w(w+2)(w^2 - 2w + 4) = 0$

$w = 0$ or $w + 2 = 0$ or $w^2 - 2w + 4 = 0$

$w = -2$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4(1)(4)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-12}}{2}$$

$$= \frac{2 \pm 2\sqrt{3}i}{2} = 1 \pm \sqrt{3}i$$

Solutions are

$w = 0, -2, 1 \pm \sqrt{3}i$

② $x - \frac{\sqrt{1-3x}}{2} = 0$

$(2x)^2 = (\sqrt{1-3x})^2$

$4x^2 = 1 - 3x$

$4x^2 + 3x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{9 - 4(4)(-1)}}{2(4)}$$

$$x = \frac{-3 \pm \sqrt{25}}{8}$$

$$= \frac{-3 \pm 5}{8} = -1 \text{ or } \frac{1}{4}.$$

check: $(-1) - \frac{\sqrt{1 - 3(-1)}}{2} = -1 - \frac{2}{2}$

$= -2$

extraneous!

$(1/4) - \frac{\sqrt{1 - 3(1/4)}}{2} = \frac{1}{4} - \frac{1}{4} = 0$

One solution, $x = 1/4$.

(3) $\dfrac{1}{x} - \dfrac{2}{\sqrt{2x+1}} = 0$

$\left(\dfrac{1}{x}\right)^2 = \left(\dfrac{2}{\sqrt{2x+1}}\right)^2$

$\dfrac{1}{x^2} = \dfrac{4}{2x+1}$

$2x+1 = 4x^2$

$4x^2 - 2x - 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{2 \pm \sqrt{4-4(4)(-1)}}{2(4)}$

$= \dfrac{2 \pm \sqrt{20}}{8} = \dfrac{2 \pm 2\sqrt{5}}{8} = \dfrac{1 \pm \sqrt{5}}{4 \cancel{8}}$

check: $x = \dfrac{1+\sqrt{5}}{\cancel{8}4} = 0.809017$

$\dfrac{1}{x} = \dfrac{8}{1+\sqrt{5}}$

$\dfrac{2}{\sqrt{2x+1}} = \dfrac{2}{\sqrt{2\left(\frac{1+\sqrt{5}}{8}\right)+1}}$

$= \dfrac{2}{\left(\dfrac{\sqrt{5+\sqrt{5}}}{2}\right)} = \dfrac{4}{\sqrt{5+\sqrt{5}}}$

should be same

This is one of those tricky ones to check!
switch to decimals

$\dfrac{1}{x} = 1.23607$

$\dfrac{2}{\sqrt{2x+1}} = 1.23607$

so $x = \dfrac{1+\sqrt{5}}{4}$ is a solution.

check $x = \dfrac{1-\sqrt{5}}{4} = -0.309017$

$\dfrac{1}{x} = -3.23607$

$\dfrac{2}{\sqrt{2x+1}} = 3.23607$

not a solution!

Only on solution, $x = \dfrac{1+\sqrt{5}}{4}$

(4) $\sqrt{2x+5} + \sqrt{x+6} = 9$

$$\left(\sqrt{2x+5}\right)^2 = \left(9 - \sqrt{x+6}\right)^2$$

$$2x+5 = 81 - 18\sqrt{x+6} + (x+6)$$

$$x - 82 = -18\sqrt{x+6}$$

$$(x-82)^2 = \left(-18\sqrt{x+6}\right)^2$$

$$x^2 - 164x + 82^2 = +18^2(x+6)$$

$$x^2 - 488x + 4780 = 0$$

$$(x-10)(x-478) = 0$$

$$x = 10 \quad \text{or} \quad x = 478$$

<u>Check $x=10$:</u> $\sqrt{2(10)+5} + \sqrt{10+6} = 5 + 4 = 9$ OK!

<u>Check $x=478$:</u> $\sqrt{2(478)+5} + \sqrt{478+6} = \sqrt{961} + \sqrt{484}$

$= 31 + 22$

$= 53 \neq 9$. Extraneous.

There is only one solution, $x=10$.

(5) $(s-1)^{-1/2} = 2$

$\left[(s-1)^{-1/2}\right]^{-2} = 2^{-2}$

$s-1 = \frac{1}{4}$

$s = 1+\frac{1}{4} = 5/4.$

check:

$(5/4-1)^{-1/2} = (1/4)^{-1/2}$

$= 4^{1/2}$

$= 2$ OK.

(6) Let $\frac{2c-3}{5} = x$

$x^2+2x=8$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x+4=0$ or $x-2=0$

$x=-4$ $\quad$ $x=2$

$\frac{2c-3}{5}=-4$ $\quad$ $\frac{2c-3}{5}=2$

$2c-3=-20$ $\quad$ $2c-3=10$

$2c=-17$ $\quad$ $2c=13$

$c=-17/2$ $\quad$ $c=13/2$

check: $\left(\frac{2(-17/2)-3}{5}\right)^2+2\left(\frac{2(-17/2)-3}{5}\right) = 16-8=8$ OK.

$\left(\frac{2(13/2)-3}{5}\right)^2+2\left(\frac{2(13/2)-3}{5}\right) = 4+4=8$ OK.

Two solutions $c=-17/2$ and $13/2$.

⑦ Let $w = \dfrac{1}{5x-1}$.

$w^2 + w = 12$

$w^2 + w - 12 = 0$

$(w+4)(w-3) = 0$

$w = -4$ or $w = 3$

$\dfrac{1}{5x-1} = -4$ or $\dfrac{1}{5x-1} = 3$

$1 = -4(5x-1)$ $\quad$ $1 = 3(5x-1)$

$1 = -20x + 4$ $\quad$ $1 = 15x - 3$

$-3 = -20x$ $\quad$ $4 = 15x$

$x = \dfrac{3}{20}$ $\quad$ $x = \dfrac{4}{15}$

check: $\dfrac{1}{(5(\frac{3}{20})-1)^2} + \dfrac{1}{5(\frac{3}{20})-1} = \dfrac{1}{(1/16)} + \dfrac{1}{(-1/4)} = 16 - 4 = 12$ OK

$\dfrac{1}{(5(\frac{4}{15})-1)^2} + \dfrac{1}{5(\frac{4}{15})-1} = \dfrac{1}{(1/9)} + \dfrac{1}{(1/3)} = 9 + 3 = 12$ OK.

Two solutions $x = 3/20$ and $4/15$.

8. $|x^2+5x| = |3-x^2|$

$x^2+5x = 3-x^2$ or $x^2+5x = -(3-x^2)$

$2x^2+5x-3=0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-5 \pm \sqrt{25-4(2)(-3)}}{2(2)}$

$= \dfrac{-5 \pm 7}{4}$

$= -3$ or $\frac{1}{2}$

$x^2+5x = -3+x^2$

$5x = -3$

$x = -3/5$

checks

$\left|(-3)^2+5(-3)\right| \stackrel{?}{=} \left|3-(-3)^2\right|$

$|6| = |-6|$ True

$\left|(1/2)^2+5(1/2)\right| \stackrel{?}{=} \left|3-(1/2)^2\right|$

$\left|\frac{11}{4}\right| = \left|\frac{11}{4}\right|$ True

$\left|(-3/5)^2+5(-3/5)\right| \stackrel{?}{=} \left|3-(-3/5)^2\right|$

$\left|\frac{9}{25}-3\right| = \left|3-\frac{9}{25}\right|$ True.

Three solutions $x = -3, 1/2, -3/5$.

⑨ Let x be time when distance between Nanc & Ed is 14 miles greater than distance between Nanc & Will.

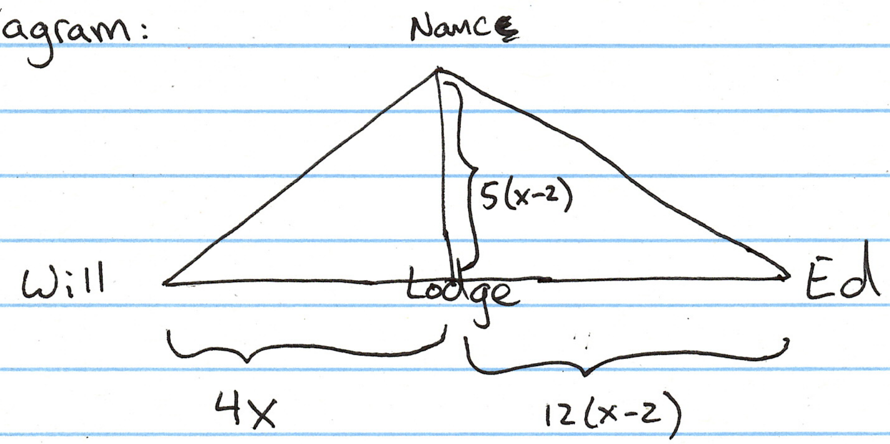
(# hours after 8am!)

After x hours, Will has ~~gone~~ been hiking for x hours and has travelled $4x$ miles.

After x hours, Nanc has been hiking for $x-2$ hours and has travelled $5(x-2)$ miles.

After x hours, Ed has been hiking for $x-2$ hours and has travelled $12(x-2)$ miles.

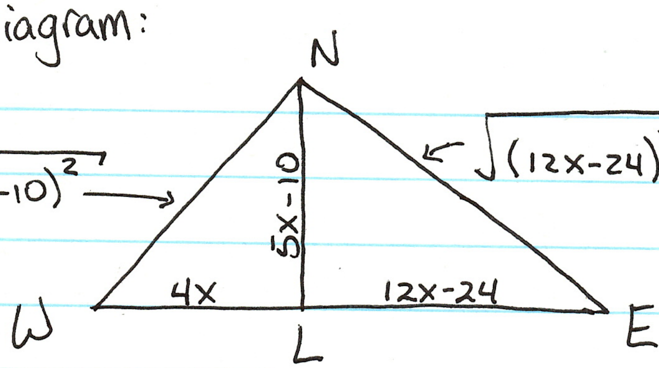
Diagram:



Use Pythagorean Theorem to get distance between Will & Nanc and Nanc & ED.

Clean up diagram:

$|NW| = \sqrt{(4x)^2 + (5x-10)^2}$ →



← $\sqrt{(12x-24)^2 + (5x-10)^2} = |NE|$

We want $|NE| = |NW| + 14$

Whoops! Did not read diagram correctly.

$$\cancel{\sqrt{(4x)^2 + (5x-10)^2} = \sqrt{(12x-24)^2 + (5x-10)^2} + 14}$$

Solution to this is time (in hours past 8am) when $|NE| = |NW| + 14$.

$$\sqrt{(12x-24)^2 + (5x-10)^2} = \sqrt{(4x)^2 + (5x-10)^2} + 14$$

$$(12x-24)^2 + (5x-10)^2 = \left(\sqrt{16x^2 + (5x-10)^2} + 14\right)^2$$

$$(144x^2 - 576x + 576) + (100 - 100x + 25x^2) = 41x^2 - 100x + 296 + 28\sqrt{16x^2 + (5x-10)^2}$$

$$169x^2 - 676x + 676 = 41x^2 - 100x + 296 + 28\sqrt{16x^2 + (5x-10)^2}$$

Who am I kidding, I would not ask you to solve this by hand. But setting up the equation was fun!

FYI, the solution to the equation is $x = 5$.

So 1pm (5 hours after 8am) is when everyone is the correct distance apart.

Here is how the computer algebra system *Mathematica* can be used to solve the equation:

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Solve[Sqrt[(12 x - 24)^2 + (5 x - 10)^2] == Sqrt[(4 x)^2 + (5 x - 10)^2] + 14, x]
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$$\left\{\{x \to 5\}, \left\{x \to \frac{1}{64}\left(53 - \sqrt{1401}\right)\right\}\right\}$$

The second value is extraneous.

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N[%]
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{{x → 5.}, {x → 0.243282}}
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