Questions

1. Sketch the graph of $f(x) = -e^{-3x} + 4$ by transforming $g(x) = e^x$. From your sketch, evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$, and state the domain and range of f.

2. Sketch the graph of $f(x) = -e^{12x} - 3$ by transforming $g(x) = e^x$. From your sketch, evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$, and state the domain and range of f.

3. A Bank pays 8% annual interest compounded monthly. How much will a deposit of \$5000 amount to after 6 years?

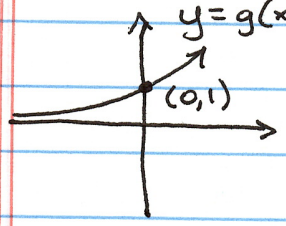
4. A Bank pays 8% annual interest compounded continuously. How much will a deposit of \$5000 amount to after 6 years?

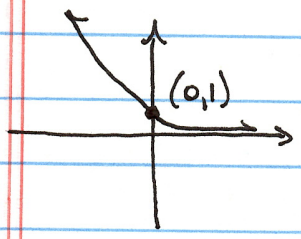
5. Solve each equation.

(a) $\left(\frac{2}{3}\right)^x = \frac{9}{4}$.

(b) $\left(\frac{4}{9}\right)^x \left(\frac{8}{27}\right)^{1-x} = \frac{2}{3}$.

(1) $f(x) = -e^{-3x} + 4$



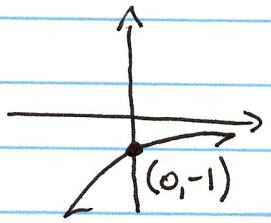


$y = g(-3x) = e^{-3x}$

inside → horizontal

reflect about y-axis,

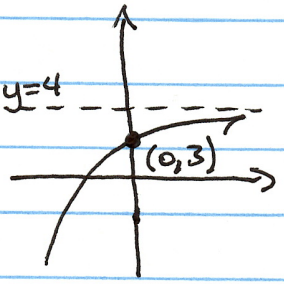
horizontal compression by a factor of 3.



$y = -g(-3x) = -e^{-3x}$

outside → vertical

reflect about x-axis.



$y = -g(-3x) + 4 = -e^{-3x} + 4$

outside → vertical

shift up 4 units

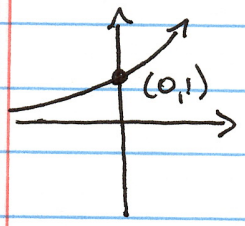
$\lim_{x \to \infty} [f(x)] = 4$ horizontal asymptote.

$\lim_{x \to -\infty} [f(x)] = -\infty$

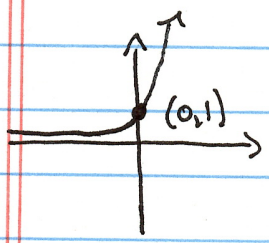
domain $x \in \mathbb{R}$

range $y \in (-\infty, 4)$.

(2) $f(x) = -e^{12x} - 3$



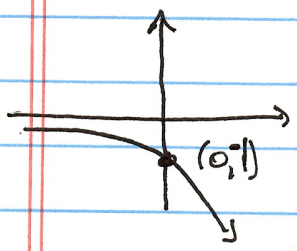
$y = g(x) = e^x$



$y = g(12x) = e^{12x}$

inside → horizontal

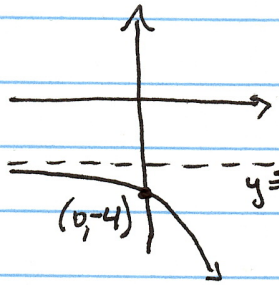
horizontal compression by a factor of 12.



$y = -g(12x) = -e^{12x}$

outside → vertical

reflect about x-axis



$y = -g(12x) - 3$

outside → vertical

shift down 3 units

$\lim_{x \to \infty} [f(x)] = -\infty$

$\lim_{x \to -\infty} [f(x)] = -3$ horizontal asymptote

domain $x \in \mathbb{R}$

range $y \in (-\infty, -3)$.

③ Compound interest $A = P\left(1+\frac{r}{n}\right)^{nt}$

or $A = P(1+i)^M$

A is accumulated amount

$r = 8\% = 0.08$ is annual interest rate

$n = 12$ is number of compounding periods per year

$i = 0.08/12 \approx 0.0066667$

$M = 12 \cdot 6 = 72$ compounding periods to get to 6 years

$P = \#5,000$ is the principal.

$A = P(1+i)^M$

$= \#5000(1+0.0066667)^{72}$

$= \#8067.51$

④ Continuous Compounding $A = Pe^{rt}$

A is accumulated amount

$r = 8\% = 0.08$ is annual interest rate

$t = 6$ years

$P = \#5000$

$A = Pe^{rt}$

$= \#5000\, e^{0.08 \cdot 6}$

$= \#8080.37$

(5a) $\left(\frac{2}{3}\right)^x = \frac{9}{4}$

Note: Once we have logarithms and rules of logarithms we can solve for x a different way.

Get bases to be the same

$\left(\frac{2}{3}\right)^x = \left(\frac{3}{2}\right)^2$

$\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^{-2}$ bases same, so exponents must be equal. $x = -2$

(5b) $\left(\frac{4}{9}\right)^x \left(\frac{8}{27}\right)^{1-x} = \frac{2}{3}$ Get bases the same.

$\left(\left(\frac{2}{3}\right)^2\right)^x \left(\left(\frac{2}{3}\right)^3\right)^{1-x} = \frac{2}{3}$

$\left(\frac{2}{3}\right)^{2x} \left(\frac{2}{3}\right)^{3-3x} = \frac{2}{3}$

$\left(\frac{2}{3}\right)^{2x+3-3x} = \left(\frac{2}{3}\right)$ bases same, so exponents must be equal.

$\left(\frac{2}{3}\right)^{3-x} = \left(\frac{2}{3}\right)$

$3 - x = 1$

$x = 2.$