Questions

1. Rewrite each expression as a single logarithm.

(a) $\log(x^2 - 4) - \log(x - 2)$

(b) $\ln(x^3) + \ln(x - 1) - \ln(x + 1)$

(c) $\frac{1}{2}\ln(x) - \frac{1}{3}\ln(y) + \ln(z)$

2. Solve each equation.

(a) $2\left(1 + \frac{r}{12}\right)^{360} = 8$

(b) $(1.025)^{360t} = 8000$

3. Solve $A = P\left(1 + \frac{r}{n}\right)^{nt}$ for r.

4. Solve $A = P\left(1 + \frac{r}{n}\right)^{nt}$ for t.

5. Solve $A = Pe^{rt}$ for t.

6. Solve $A = A_0e^{rt}$ for r.

7. Find f^{-1} given $f(x) = 2e^{-x+3} + 4$. Check your answer by computing $f \circ f^{-1}$.

8. Find f^{-1} given $f(x) = 4 - \ln(-x + 3)$. Check your answer by computing $f \circ f^{-1}$.

9. Write the expression $\log_{12}(45)$ in base e with a natural logarithm.

10. What is the domain of $f(x) = \ln(x(x - 1))$?

11. Given $f(x) = -3\ln(2x) + \ln(x^5)$, what is the domain of f? What is the range of f^{-1}? Find the algebraic expression for the inverse function $f^{-1}(x)$.

12. Given $f(x) = \ln(\sqrt{x})$, $g(x) = e^{x/4}$, and $h(x) = x^2$. Find the composition $(f \circ g \circ h)(x)$ and simplify as much as possible. Your final answer should **not** have exponentials and logarithms in it.

1a) $\log(x^2-4) - \log(x-2) = \log\left(\frac{x^2-4}{x-2}\right)$

$$= \log\left(\frac{(x-2)(x+2)}{x-2}\right)$$

$$= \log(x+2) \qquad x-2 \neq 0 \rightarrow x \neq 2.$$

b) $\ln(x^3) + \ln(x-1) - \ln(x+1) = \ln\left(\frac{x^3(x-1)}{x+1}\right)$

c) $\frac{1}{2}\ln(x) - \frac{1}{3}\ln(y) + \ln(z) = \ln(x^{1/2}) - \ln(y^{1/3}) + \ln(z)$

$$= \ln\left(\frac{x^{1/2}}{y^{1/3}} \cdot z\right)$$

2a) $2\left(1 + \frac{r}{12}\right)^{360} = 8$

$$\left[\left(1 + \frac{r}{12}\right)^{360}\right]^{1/360} = [4]^{1/360}$$

$$1 + \frac{r}{12} = 4^{1/360}$$

$$r = 12\left(4^{1/360} - 1\right).$$

b) $(1.025)^{360t} = 8000$

$$\ln\left[(1.025)^{360t}\right] = \ln[8000]$$

$$360t \ln[1.025] = \ln[8000]$$

$$t = \frac{\ln[8000]}{360 \ln[1.025]}$$

(3) $A = P\left(1+\frac{r}{n}\right)^{nt}$ Solve for r.

$$\left[\frac{A}{P}\right]^{1/nt} = \left[\left(1+\frac{r}{n}\right)^{nt}\right]^{1/nt}$$

$$\left(\frac{A}{P}\right)^{1/nt} = 1+\frac{r}{n}$$

$$n\left(\left(\frac{A}{P}\right)^{1/nt} - 1\right) = r$$

(4) $A = P\left(1+\frac{r}{n}\right)^{nt}$ Solve for t.

$$\ln\left[A/P\right] = \ln\left[\left(1+r/n\right)^{nt}\right]$$

$$\ln\left[A/P\right] = nt \ln\left[1+\frac{r}{n}\right]$$

$$t = \frac{\ln\left[A/P\right]}{n \ln\left[1+\frac{r}{n}\right]}$$

(5) $A = Pe^{rt}$ Solve for t.

$$\ln\left[A/P\right] = \ln\left[e^{rt}\right]$$

$$\ln\left[A/P\right] = rt$$

$$t = \frac{\ln\left[A/P\right]}{r}.$$

⑥ $A = A_0 e^{rt}$ solve for r.

$$\ln\left[A/A_0\right] = \ln\left[e^{rt}\right]$$

$$\ln\left[A/A_0\right] = rt$$

$$t = \frac{\ln\left[A/A_0\right]}{r}$$

⑦ $f(x) = 2e^{-x+3} + 4.$

$$y = 2e^{-x+3} + 4$$

interchange x and y

$$x = 2e^{-y+3} + 4$$

solve for y

$$\frac{x-4}{2} = e^{-y+3}$$

$$\ln\left[\frac{x-4}{2}\right] = \ln\left[e^{-y+3}\right]$$

$$\ln\left[\frac{x-4}{2}\right] = -y+3$$

$$y = f^{-1}(x) = 3 - \ln\left[\frac{x-4}{2}\right]$$

check: $f(f^{-1}(x)) = f\left(3 - \ln\left[\frac{x-4}{2}\right]\right)$

$$= 2e^{-\left(3-\ln\left[\frac{x-4}{2}\right]\right)+3} + 4$$

$$= 2e^{\ln\left[\frac{x-4}{2}\right]} + 4$$

$$= 2\left(\frac{x-4}{2}\right) + 4$$

$$= x-4+4$$

$$= x \quad \checkmark$$

⑧ $y = 4 - \ln(-x+3)$

interchange x and y: $x = 4 - \ln(-y+3)$

$\ln(-y+3) = 4 - x$

Solve for y:

$e^{\ln(-y+3)} = e^{4-x}$

$-y+3 = e^{4-x}$

$y = 3 - e^{4-x} = f^{-1}(x)$.

<u>check</u>

$f(f^{-1}(x)) = f(3 - e^{4-x})$

$= 4 - \ln(-(3 - e^{4-x}) + 3)$

$= 4 - \ln(e^{4-x})$

$= 4 - (4-x)$

$= x$ ✓

⑨ $y = \log_{12}(45) \longrightarrow 12^y = 45$

$\ln(12^y) = \ln(45)$

$y\ln(12) = \ln(45)$

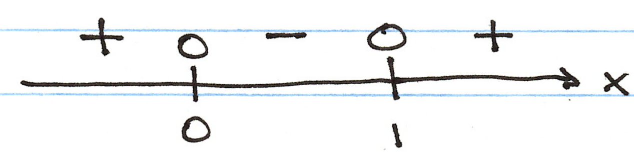
$y = \dfrac{\ln(45)}{\ln(12)} = \log_{12}(45)$

⑩ $f(x)$ is defined if $x(x-1) > 0$. Use sign chart to simplify.

<u>zeros</u> $\left.\begin{array}{l} x=0 \\ x=1 \end{array}\right\}$ multiplicity 1, so $x(x-1)$ will change sign

<u>End Behaviour</u> As $x \to \infty$ $x(x-1) \to x(x) = x^2 > 0$.

Note $x \to -\infty$ $x(x-1) \to x(x) = x^2 > 0$.



sign chart for $x(x-1)$.

Domain of $f(x) = \ln(x(x-1))$ is $x \in (-\infty, 0) \cup (1, \infty)$.

(11) $f(x) = -3\ln(2x) + \ln(x^5)$.

Domain of f: $2x > 0$ and $x^5 > 0$

which simplified $x > 0$ and $x > 0$.

Both are true when $x > 0$.

Domain of f is $x > 0$, $x \in (0, \infty)$.

Range of f^{-1} is the domain of f, $y > 0$, $y \in (0, \infty)$.

Get f^{-1}

$$y = -3\ln(2x) + \ln(x^5)$$

interchange x and y

$$x = -3\ln(2y) + \ln(y^5)$$

Solve for y

$$x = \ln((2y)^{-3}) + \ln(y^5)$$

$$x = \ln((2y)^{-3}y^5)$$

$$x = \ln\left(\frac{y^5}{(2y)^3}\right)$$

$$x = \ln\left(\frac{y^5}{8y^3}\right)$$

$$x = \ln\left(y^2/8\right)$$

$$e^x = e^{\ln(y^2/8)}$$

$$e^x = y^2/8$$

$$y = \pm(8e^x)^{1/2}$$

$$= \pm\sqrt{8}\,e^{x/2}$$

we have to pick one of these. Since range of f^{-1} is $y > 0$, choose positive root, $y = f^{-1}(x) = \sqrt{8}\,e^{x/2}$.

⑫ $(f \circ g \circ h)(x) = f(g(h(x)))$

$$= f(g(x^2))$$

$$= f(e^{x^2/4})$$

$$= \ln\left(\sqrt{e^{x^2/4}}\right)$$

$$= \ln\left(\left(e^{x^2/4}\right)^{1/2}\right)$$

$$= \ln\left(e^{x^2/8}\right) \quad \underline{\underline{\text{or}}} \quad = \frac{1}{2}\ln\left(e^{x^2/4}\right)$$

$$= \frac{x^2}{8} \qquad\qquad = \frac{1}{2}\left(\frac{x^2}{4}\right)$$

$$= \frac{x^2}{8}.$$