Math 1012 Precalculus I Functions Test 4
Practice

The Rules of Exponentials and Logarithms

1. Choose the correct rules for logarithms and exponentials from the list below.

$\ln(A/B) = \dfrac{\ln(A)}{\ln B}$ . . . . T F

$e^{a-b} = \dfrac{e^a}{e^b}$ . . . . T F

$B\ln(A) = [\ln(A)]^B$ . . . . T F

$\ln(A^B) = B\ln(A)$ . . . . T F

$e^{a+b} = e^a e^b$ . . . . T F

$e^{a+b} = e^a + e^b$ . . . . T F

$\ln(A) - \ln(B) = \ln(A/B)$ . . . . T F

$e^{\ln A} = A$ . . . . T F

$e^{a-b} = e^{a/b}$ . . . . T F

$\ln(A) = A\ln$ . . . . T F

$\ln(e) = 1$ . . . . T F

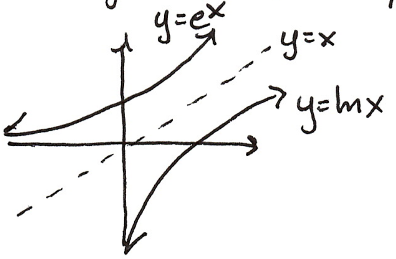
$\ln(e^A + e^B) = A + B$ . . . . T F

$\ln(A) + \ln(B) = \ln(AB)$ . . . . T F

$\ln(e^A) = A$ . . . . T F

$\ln(A+B) = \ln(A) + \ln(B)$ . . . . T F

① Write $y=f(x)$. Interchange x and y, $x=f(y)$. Solve for $y=f^{-1}(x)$. Note Domain of f is range of f^{-1} (can help if you need to choose $\pm\sqrt{\ }$).

② 

$y=e^x$ has horizontal asymptote since $\lim_{x\to-\infty}[e^x]=0$
Domain $x\in(-\infty,\infty)$
$y=\ln(x)$ has vertical asymptote since $\lim_{x\to0^+}[\ln(x)]=-\infty$.
Domain $x\in(0,\infty)$.

③ Let y equal logarithm you are converting. Convert to exponential form using

$$\log_b(c)=y \longleftrightarrow b^y=c$$

Take logarithm of new base, use $\log_w(b^y)=y\log_w(b)$ to solve for y.

④ Not the same. $y=\log(x^2)$ domain $x^2>0 \to x\in(-\infty,0)\cup(0,\infty)$
$y=2\log(x)$ domain $x>0 \to x\in(0,\infty)$.
$\log(x^2)=2\log|x|$.

⑤ Use logarithm rules to write equation as $\ln(A)=B$. Then take exponentials of both sides of equation $f(x)=e^B$. Solve for x.

⑥ Use exponent rules to write equation as $e^{f(x)}=B$. Then take logarithm of both sides of equation $f(x)=\ln(B)$. Solve for x.

Be on the lookout for substitution to make equation quadratic, isolate square root to square, etc.

ie) $e^{2x}+e^x-2=0 \to (e^x)^2+e^x-2=0$ let $u=e^x$.

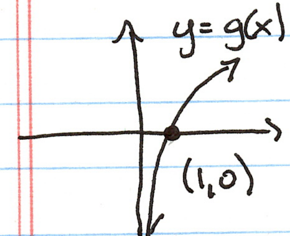
(7) $f(x) = -2\ln(-4x)$

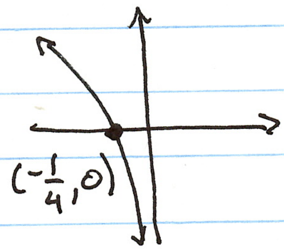
Domain: $-4x > 0$

$x < 0$.

Range of f^{-1} is $y < 0$.

sketch f by graphical transformations

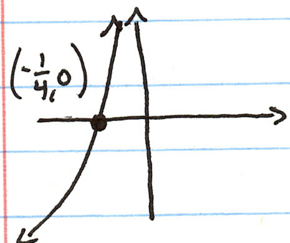




$y = g(-4x) = \ln(-4x)$

inside $\rightarrow$ horizontal

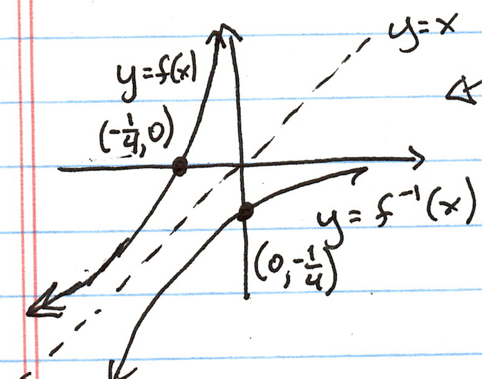
reflect about y-axis, and compress by factor of 4.



$y = -2g(-4x) = -2\ln(-4x)$

outside $\rightarrow$ vertical

reflect about x-axis, stretch vertically by factor of 2.



$\leftarrow$ Reflect about line $y=x$ to sketch $f^{-1}(x)$.

Get inverse algebraically:

$x = -2\ln(-4y)$

$\frac{x}{-2} = \ln(-4y)$

$e^{-x/2} = -4y \rightarrow y = f^{-1}(x) = \frac{-1}{4} e^{-x/2}$.

check $f(f^{-1}(x)) = f\left(-\frac{1}{4}e^{-x/2}\right)$

$= -2\ln\left(-4\left(-\frac{1}{4}e^{-x/2}\right)\right)$

$= -2\ln\left(e^{-x/2}\right)$

$= -2\left(\frac{-x}{2}\right)$

$= x$ ✓

(8) $f(x) = -2\ln(-6x) + \ln(x^4)$

Domain: $-6x > 0 \rightarrow x < 0$ $\quad\rightarrow$ both true if $x<0$.

$x^2 > 0 \rightarrow x > 0$ or $x < 0$ $\quad$ Domain f is $x<0$.

Range f^{-1} is $y<0$.

$x = -2\ln(-6y) + \ln(y^4)$ $\quad$ solve for y.

$x = \ln((-6y)^{-2}) + \ln(y^4)$

$x = \ln((-6y)^{-2} y^4)$

$x = \ln\left(\dfrac{y^4}{36y^2}\right)$

$x = \ln\left(\dfrac{y^2}{36}\right)$

$e^x = \dfrac{y^2}{36} \rightarrow y = \pm\sqrt{36e^x}$ $\qquad f^{-1}(x) = -6\sqrt{e^x} = -6e^{x/2}$

choose $-\sqrt{}$ since range of f^{-1} is $y<0$.

<u>Check</u> $f(f^{-1}(x)) = f(-6e^{x/2})$

$= -2\ln(-6(-6e^{x/2})) + \ln((-6e^{x/2})^4)$

$= -2\ln(36e^{x/2}) + \ln(+6^4e^{2x})$

$= \ln((36e^{x/2})^{-2}) + \ln(6^4e^{2x})$

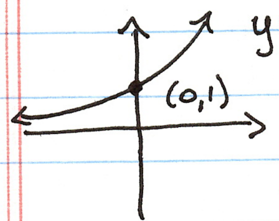
$= \ln\left(\dfrac{6^4e^{2x}}{36^2e^x}\right)$

$= \ln(e^{2x-x})$

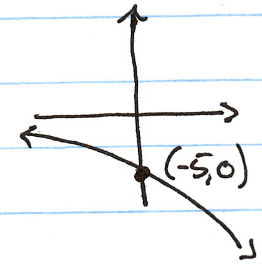
$= \ln(e^x)$

$= x.$ ✓

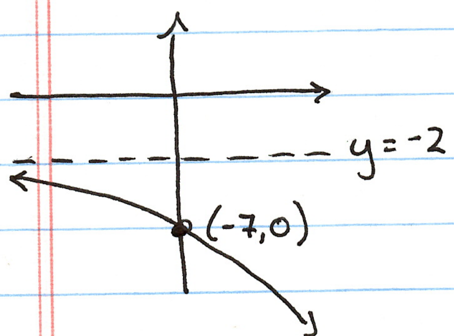
(9) $f(x) = -5e^x - 2$



$y = e^x = g(x)$



$y = -5g(x) = -5e^x$
outside → vertical
reflect about x-axis
and vertical stretch by
factor of 5.



$y = -5g(x) - 2 = -5e^x - 2$
outside → vertical
shift down 2 units.

Domain $x \in \mathbb{R}$ Range $y \in (-\infty, -2)$
no vertical asymptotes. Horizontal Asymptote $y = -2$
never increasing. Decreasing on $x \in (-\infty, \infty)$.

$$\lim_{x \to -\infty} \left[-5e^x - 2\right] = -2 \qquad \lim_{x \to \infty} \left[-5e^x - 2\right] = -\infty.$$

(10)

doubles every 24 months

$P(0) = 200$ time in months

$P(24) = 2 \cdot 200$

$P(2 \cdot 24) = 2^2 \cdot 200$

$P(3 \cdot 24) = 2^3 \cdot 200$

$P(4 \cdot 24) = 2^4 \cdot 200$

$\vdots$ look for pattern

$P(t) = 2^{t/24} \cdot 200$

Population is $10000 = 2^{t/24} \cdot 200$

$50 = 2^{t/24}$

$\ln(50) = \ln\left(2^{t/24}\right)$

$\ln(50) = \frac{t}{24} \ln(2)$

$\rightarrow t = \dfrac{24 \ln(50)}{\ln(2)}$ months.

(11)

$$\frac{R}{P} = \frac{i}{1-(1+i)^{-nt}}$$

$$1-(1+i)^{-nt} = \frac{Pi}{R}$$

$$1 - \frac{Pi}{R} = (1+i)^{-nt}$$

$$\ln\left(1-\frac{Pi}{R}\right) = \ln\left[(1+i)^{-nt}\right]$$

$$\ln\left(1-\frac{Pi}{R}\right) = -nt \ln\left[(1+i)\right]$$

$$t = \frac{\ln\left(1-\frac{Pi}{R}\right)}{-n \ln(1+i)}$$

(12) $\left[2^x - 2^{-x} = 12\right] \cdot 2^x$

$(2^x)^2 - 1 = 12(2^x)$

$(2^x)^2 - 12(2^x) - 1 = 0$

Let $u = 2^x$.

$u^2 - 12u - 1 = 0$

$\cancel{(u+3)(u-4)=0}$ WRONG!

$u = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{12 \pm \sqrt{144-4(1)(-1)}}{2(1)}$

$= \dfrac{12 \pm \sqrt{148}}{2} = 6 \pm \sqrt{37}$

Get x:

$2^x = 6 + \sqrt{37}$

$x = \log_2(6+\sqrt{37})$

or $x = \dfrac{\ln(6+\sqrt{37})}{\ln(2)}$.

$2^x = 6 - \sqrt{37}$

no solution in Real Numbers.

13 $D = D_0 e^{kt}$ $\qquad D_0 = 350 - 40 = 310$

$D(4) = 350 - 170 = 310 e^{k(4)}$ solve for k.

$$\frac{180}{310} = e^{4k}$$

$$\ln\left(\frac{18}{31}\right) = 4k \rightarrow k = \frac{1}{4}\ln\left(\frac{18}{31}\right)$$

$$D = 310 e^{\frac{1}{4}\ln\left(\frac{18}{31}\right)t}$$

Turkey is done when $D(t) = 350 - 185 = 165$

$$165 = 310 e^{\frac{1}{4}\ln\left(\frac{18}{31}\right)t}$$

$$\frac{165}{310} = e^{\frac{1}{4}\ln\left(\frac{18}{31}\right)t}$$

$$\ln\left(\frac{165}{310}\right) = \frac{1}{4}\ln\left(\frac{18}{31}\right)t$$

$$t = \frac{4\ln\left(\frac{165}{310}\right)}{\ln\left(\frac{18}{31}\right)}$$ hours.

(14) $\ln(x) - \ln(x+1) = \ln(x+3) - \ln(x+5)$

$$\ln\left(\frac{x}{x+1}\right) = \ln\left(\frac{x+3}{x+5}\right)$$

$$\frac{x}{x+1} = \frac{x+3}{x+5}$$

$$x(x+5) = (x+3)(x+1)$$

$$x^2+5x = x^2+4x+3$$

$$x = 3$$

Check $\ln(3) - \ln(4) = \ln(6) - \ln(8)$

$$\ln(3/4) = \ln(6/8) \checkmark$$

(15) $e^{3\ln(x^2) - 2\ln(x)} = 16$

$$e^{\ln(x^6) - \ln(x^2)} = 16$$

$$e^{\ln(x^6/x^2)} = 16$$

$$x^4 = 16$$

$$x = (16)^{1/4} = 2$$

(16) $\log_{10}(x+1) - \log_{10}(x) = 3$

$$\log_{10}\left(\frac{x+1}{x}\right) = 3$$

$$10^3 = \frac{x+1}{x}$$

$$1000x = x+1$$

$$999x = 1$$

$$x = \frac{1}{999}.$$