Section 2.6

Designing Efficient Snow Plow Routes: A Service-Learning Project

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Abstract: We describe our experience incorporating service-learning into a combinatorics and graph theory course. In particular, we present a discrete mathematical model which we used to design efficient snow plow routes for Morris, Minnesota. We also discuss several issues of service-learning within the course as well as beyond the classroom.

Introduction

In 1996, three mathematics faculty members of the University of Minnesota, Morris were awarded a SEAMS (Science, Engineering, Architecture, Mathematics, Computer Science) Grant from the Minnesota Campus Compact to incorporate service-learning into a mathematics course and two statistics courses. Part of the grant included the support of a coordinator, Mr. Ben Winchester, who took care of all administrative tasks ranging from setting up meetings between the students and the city officials to handling paper work involving expenditures incurred and surveys for assessment purposes. One reason that this service-learning project went smoothly in the course is the accommodating help of the coordinator.

Our objective in this paper is to discuss one of the projects that was done in the math course, Discrete & Combinatorial Mathematics, as well as touch upon a few issues to think about when incorporating service-learning into mathematics. This math course carries 4 credits, has about 12 students and, under the quarter and semester systems it lasts for 10 and 15 weeks, respectively. The course objectives are to help students:

- to learn the concepts of logic, proofs, and the language of mathematics;
- to understand the notions of discrete functions, sets, counting processes, and combinatorics;
- to learn the concepts of graph theory;
- to be aware of the connections between discrete mathematics and other math-related courses; and
- to be aware of the plethora of applications of discrete and combinatorial mathematics within and beyond the scope of academia.

The contents of this course include topics found in most discrete mathematics text books and topics in classical combinatorics and graph theory with applications.

It is mandatory for this course to have a course project where students find a real-world application of the material covered, write a paper on it and give an oral presentation to the class at the end of the course. When we incorporated service-learning, the main difference was that the particular projects were chosen to be of value to various community agencies within the city or county.

In the first section below, we describe a particular project which was of interest and beneficial value to the City of Morris, a small community of about 5600 people located in rural west central Minnesota. We also give the detailed mathematical model we used to solve a specific problem posed by the city. One of the objectives of the detailed description of the model is to illustrate the fact that in a process which is rich in both useful and useless data, it behooves all parties concerned to maximize their efforts at communications. In the next section, we summarize how we conducted the course, including how students worked on the projects, how they were graded, and how they communicated with the community agencies. In the last section, we discuss the rewards we reaped as well as challenges that we encountered throughout the process.

A Mathematical Model

There were several questions of interest that came from various community agencies, such as: What are the traffic-flow patterns around the city and its vicinity? How efficiently can we plow the streets in the city right after a snow storm? How do we group the city and its vicinity into several zones so that traffic crossing between zones is minimized? There is no room in this paper to describe all the models used to answer all the questions, but in this section, we will discuss one specific problem in detail so that you will have a rough idea of how elaborate the analysis could be based on just one specific question. At the end of this section, we list a few other common community-related projects that can also be modeled in a similar way.

Description of Project

We were asked to determine and design a cost-effective way to plow the streets and alleys so that each of the different types of snow plows and sanding trucks could traverse its group of streets and return to its depot. Pertinent information and constraints had to be considered. These include the following constraints: During a snow emergency period, there are a few roads that must be cleared first before others; costs are assumed to be proportional to the distance traveled by the snow plows; also, the city has five different classifications of plows (motor grater, reversible plow on a truck, one-way plow on a truck, a V-plow with a sander, a loader), and it has two motor graters and only one of each of the other types. Table 2.6.1 and the map in Figure 2.6.1 give a summary of the aforementioned information and constraints.

Table 2.6.1: Classifications of plows and the regions of the city

Plow type	Region Plowed
Motor grater (1)	Roads on the south-west of the city
Motor grater (2)	Roads on the north-east of the city
Reversible	Roads in south-east part of the city
One-way	Alleys in south-west of the city
V-plow	Alleys in north-east of the city
Loader	Parking lots and airport

SNOW EMERGENCY ROUTES

During snow storms before a "Snow Removal Alert" is declared and immediately after a snow storm certain snow alert route will be plowed. These streets are highlighted on the map below.

Figure 2.6.1 Annotated map of the city of Morris

A Bit of Graph Theory

The main concepts from discrete and combinatorial mathematics that were used to solve this problem come from graph theory and applications. To make this paper self-contained, we will give a brief introduction to graph theory and touch upon terminology that pertains to this project. For an in-depth coverage of graph theory, good references are Bondy and Murty [2] and West [3].

A *graph*, denoted, $G=(V,E)$, is a pair (V,E) where V is a set of *vertices* and E is a set of two-element subsets of V called *edges* ($E \subseteq \{ (i,j): i,j \in V \}$). We will now define a few structural terms in a graph, $G=(V,E)$. Refer to Figure 2.6.2 for an example.

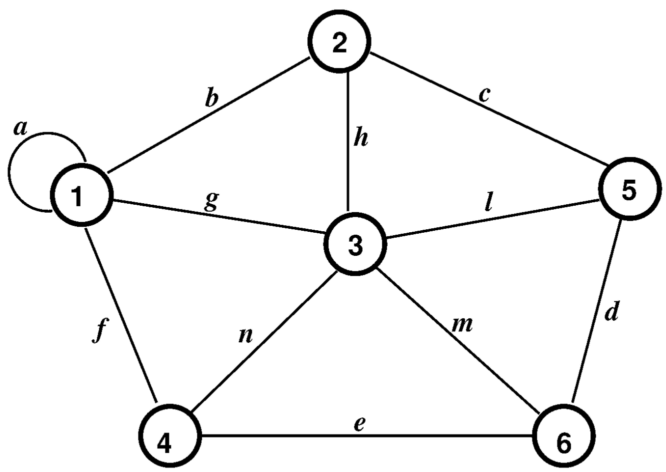

Figure 2.6.2 An example of a graph *G*=(*V*,*E*)

For $v \in V$, an edge of the type (v,v) is called a *loop*. If $u,v \in V$ where $u \neq v$ then (u,v) and (u,v) are called *parallel edges*. The *degree* of a vertex is the number of edges incident to the vertex, with a loop counting twice. A graph G is *simple* if it contains no loops and no parallel edges. A *walk from vertex v_0 to vertex v_k* is a finite sequence $v_0,e_1,v_1,e_2,v_2,\ldots,e_k,v_k$, where each $v_i \in V$ for $i=0,1,2,\ldots,k$ and each $e_i \equiv (v_{i-1},v_i) \in E$ for $i=1,2,\ldots,k$. A *trail from vertex v_0 to vertex v_k* is a walk from vertex v_0 to vertex v_k which contains no repeated edges, while a *path from vertex v_0 to vertex v_k* is a trail from vertex v_0 to vertex v_k which contains no repeated vertices. Examples of a trail and a path from vertex 4 to vertex 1 in Figure 2.6.2 are $4,f,1,a,1,b,2,h,3,l,5,d,6,m,3,g,1$ and $4,e,6,m,3,g,1$, respectively. A *tour* is a trail from vertex v_0 to vertex v_0 and a *cycle* is a trail from vertex v_0 to vertex v_0 which contains no other repeated vertices. A graph $G=(V,E)$ is said to be *connected* if there exists a path between any two vertices $u,v \in V$.

Optimal Eulerization of a Graph

We will now describe how to model the *efficient snow plow routes design* as an optimization problem on a graph. First, based on each snow plow's designation area on the map in Figure 2.6.1, we construct a graph $G=(V,E)$ by creating a vertex for each possible intersection. An edge is added between two vertices (intersections) if the block between the two intersections is to be plowed. For instance, the road system in Figure 2.6.3 translates to the graph in Figure 2.6.4 with appropriate costs, which are represented by the length of the particular block.

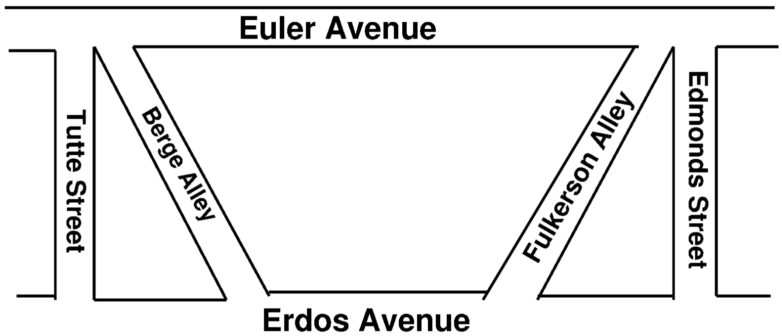

Figure 2.6.3 A snapshot of a road system (not drawn to scale).

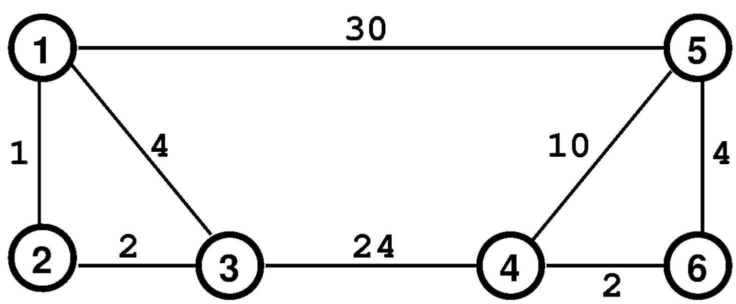

Figure 2.6.4 The graph that represents the snapshot of a road system in Figure 2.6.3, with numbers on edges corresponding to costs of traversing the edge.

Once the graph is constructed with the appropriate costs indicated on the edges, solving the city's problem is equivalent to finding a way to traverse each edge of the graph at least once, starting and returning to the same vertex, and in such a way that the total distance traversed is minimum, i.e., with as few repetitions on the edges as possible. In graph theory language, this translates to finding the optimal way to eulerize a graph if the graph is not *eulerian*. (By definition, a graph G is *eulerian* if it has an euler tour, meaning there exists a tour on G that visits all edges of G.) We found the following results useful:

Result 1. (Characterization of eulerian graphs.) *A graph $G=(V,E)$ is eulerian if and only if it is connected and all vertices have even degrees.*

Result 2. *In any graph $G=(V,E)$, there is always an even number of vertices with odd degrees.*

It is not difficult to see why Result 2 is true: when we sum the degrees of all the vertices in G we get twice the number of edges, i.e. $\sum_{v \in V} d_G(v) = 2|E|$. Thus, there must be an even number of odd terms on the left of the sum.

By and large, graphs obtained from maps of certain parts of cities are not eulerian; they usually are connected but they violate the even degree requirement. Thus, we need to find an efficient way to repeat edges so that the new graph is eulerian i.e. *optimal eulerization of a* *graph*. Even after the graph has been eulerized, or if by chance the given graph is eulerian, we still need to construct an euler tour on the graph. In summary, given a connected graph $G=(V,E)$ that represents the road map and costs c_{ij} for all $(i,j) \in E$, we first find an optimal way to duplicate edges in G so that it is eulerian, and then construct an euler tour.

The algorithm in Figure 2.6.5 gives an optimal method to eulerize a connected graph if it is not eulerian. As an illustration of the algorithm in Figure 2.6.5, refer to the graph in Figure 2.6.4, which is connected but not eulerian; vertices in the set {1,3,4,5} are of odd degrees.

Given: A connected $G = (V, E)$.
Want: A set of edges from E to duplicate so that the new graph $G' = (V, E')$ is eulerian.

Step A Check to see if G is eulerian by checking if G is both connected and all its vertices have even degrees. If it is eulerian, then we are done. Otherwise, there exists at least one vertex of odd degree.

Step B By Result 2, there is an even number of odd degree vertices. Thus, apply the Floyd-Warshall's algorithm to find the shortest path on G between every pair of the odd degree vertices. (For details of the algorithm, see Ahuja, Magnanti and Orlin [1]).

Step C Define a new graph $\bar{G} = (\bar{V}, \bar{E})$ where $\bar{V}$ is the set of vertices in V with odd degrees, and $\bar{E} = \{(i,j) : i,j \in \bar{V}\}$. And, for each $(i,j) \in \bar{E}$, assign weights,

w_{ij} = length of the shortest path between vertices i and j from Step B ; think of w_{ij} as the weight of grouping vertices i and j as a pair.

Find a *perfect matching* of $\bar{G}$ with minimum total weight, i.e. find a way to pair up the vertices in $\bar{V}$ such that the total of the pair-up weights is minimum. Since the number of vertices in $\bar{V}$ is even, we can always find a perfect matching. (See Ahuja, Magnanti and Orlin [1] for details of finding optimal perfect matching.)

Step D With the perfect matching from Step C, we duplicate edges in G in the following way: Suppose vertex u and w are paired up from Step C, we duplicate each edge on the shortest path between vertices u and w obtained in Step B. Do this for every pair of vertices in the perfect matching.

Step E Let E^d be the set of edges that were duplicated from Step D, and let $E' = E \cup E^d$. Then the new graph $G' = (V, E')$ is eulerian.

Figure 2.6.5 Algorithm for eulerizing a connected graph.

In Step B, a summary of the results of Floyd-Warshall's algorithm is given in Table 2.6.2. For Step C, since we only have four vertices of odd degrees and since we only have three possible matchings, namely, either { (1,3), (4,5) } or { (1,4), (3,5) } or { (1,5), (3,4) }, the one that yields the minimum weight is { (1,3), (4,5) } with total weight 9. For Steps D and E, the duplicated edges are those on the shortest paths between 1 and 3 and between 4 and 5, namely, E^d = { (1,2), (2,3), (4,6), (6,5) }. Clearly, after adding edges of E^d to G, the new graph, G', is eulerian.

Table 2.6.2 Results of shortest paths between odd degree vertices

i,j	Shortest path between vertices i and j	Length of the shortest path
1,3	$1 \leftrightarrow 2 \leftrightarrow 3$	3
1,4	$1 \leftrightarrow 2 \leftrightarrow 3 \leftrightarrow 4$	27
1,5	$1 \leftrightarrow 5$	30
3,4	$3 \leftrightarrow 4$	24
3,5	$3 \leftrightarrow 4 \leftrightarrow 6 \leftrightarrow 5$	30
4,5	$4 \leftrightarrow 6 \leftrightarrow 5$	6

To construct an euler tour given a graph that is eulerian, refer to any efficient algorithm based on the contructive proof of Result 1. (See Ahuja et.al. [1]). Finally, it is the euler tour constructed from G' that is interpreted as the efficient route for the snow plows.

Although the problem of designing efficient snow plow routes was posed by the public works department of the City of Morris and was a one-time project, the specific mathematical model and techniques used above are directly applicable to other problems that can also be modeled as finding an optimal eulerization of a graph. In particular, finding efficient ways to collect garbage on every block of the city, sweep the roads, do maintenance checks on an area with many railroad systems, and of course, deliver mail to all houses. Indeed, the above algorithm for finding an optimal way to eulerize a graph was proposed to solve the so-called Chinese Postman's Problem. (See Ahuja et al. [1]). One word of caution is that there are insidious consequences if one mistakenly models a real-world problem as an optimal routing problem on visiting vertices instead of one on visiting edges. The latter is efficiently solved by the technique described in this section while the former has no known efficient method to solve and probably never will.

We would also like to mention that the mathematical process, in the pedagogical sense, is similar to the description in this section even if the mathematical model used is different. For instance, in one of the posed questions on how to group the city and its vicinity into several zones so that traffic crossing between zones is minimized, once we know the number of zones, say k, then a mathematical model to use would be finding a minimum k–cut on a graph that is constructed in a similar way to the one that was created for the snow plow routes.

Administrative Process Throughout the Term

In the introductory section of this paper, we gave a description of the course and its contents. Now, we will summarize the administrative process of conducting the course, including how students worked on the projects, how they were graded, and how they communicated with the community agencies.

Prior To and As The Term Starts

Several questions of interest came from various community agencies. Before the course started, the instructor (author) spent a couple of weeks grouping the questions into two categories, namely, transportation and network flows, and network designs. The question, "what are the traffic-flow patterns around the city and its vicinity?", would fall into the first category. "How efficiently can we plow the streets in the city right after a snow storm?" would fall into the second category. "How do we group the city and its vicinity into several zones so that traffic crossing between zones is minimized?" would fall into both categories. When the course started, we divided the students randomly into three groups, each of which were randomly assigned projects within the categories mentioned above. For instance, we assigned the project described in Section 2 on designing efficient snow plow routes to a team of four students.

A course project is mandatory when we teach this course, and the project carries the same weight as one of the three within-term exams. The requirement of the course project is for students to discuss and analyze an application of a topic covered in the course; they may have to look ahead in the syllabus to browse the topics. Students usually work on these projects in pairs; each group is required to submit a written report and give an in-class 20-minute oral presentation at the end of the term. When we incorporated service-learning, we chose to do projects that were of value to the community in which we were a part.

As the Course Progressed

Each student group met regularly outside of class time to work on their project over the course of the entire term. It is advisable to have a group leader whose main job is to coordinate these meetings and keep a central working document of the written report.

The value of communication among all parties involved, i.e., the students, the instructor, the service-learning coordinator, and the community agencies, should not be underestimated. We (instructor and student groups) met on a weekly basis outside of class time and by appointment to discuss issues of, and progress on, the projects; the length of the meeting was very short at the beginning of the term and was much longer (about an hour) towards the end of the term. The instructor was a facilitator in making sure that timely progress occurred and that the analyses done and reports written were mathematically correct.

The coordinator of our SEAMS Grant was the liaison between the community agencies and the class (students and instructor). We cannot put enough emphasis on how important it is to have a liaison who is familiar with the people within the community as well as the instructor and students, and who has some knowledge of the contents of the course. Whenever we had specific questions on a piece of data or information from the agencies, the coordinator came back with responses within a couple of days or amicably pressured the community agencies for prompt responses. This is in addition to several short meetings that the coordinator set up during the first third of the course. There was an instance during the term when students felt that a face-to-face discussion on several small issues that arose in the middle of their project was necessary with a particular agency. The coordinator contacted the people of that agency and immediately scheduled a short visit to class within a few days.

At the End of the Term

Within their groups of about 4 persons each, the students did all the analyses on their own, with their instructor as an advisor and expert to bounce technical questions off. Also, they wrote their final reports and gave oral presentations to which people from Campus Compact and the community agencies were invited. Again, their instructor advised the students on the organization of their written reports and gave feedback on their trial presentations.

In terms of evaluations, a university-required course evaluation and a post service-learning evaluation from Campus Compact were given to the students at the end of the term. The only written "reflection process" on the students' part came from what they said in these evaluations.

As for grading the project, which carries about 15% of the course grade, half of the project grade is on the final report and the other half on the in-class oral presentation. The instructor was the only person who graded written reports, while the oral presentations were evaluated by all who attended. Attributes used on the evaluations of the written and oral presentations included how well students communicated the mathematical ideas, how well they demonstrated their content knowledge of their research project, the level of independent and critical thinking skills, clarity and organization, and enthusiasm. Each student within a group received the same grade on the project.

The Good News and the Challenges

In this section, we will briefly discuss a few rewards and challenges that we faced during the semester when we incorporated service-learning into Discrete and Combinatorial Mathematics. It is common for us to run into a few obstacles whenever we try to add a new aspect into our pedagogical styles, which we are very familiar and comfortable with. Since the idea of incorporating service-learning into our curriculum was a rather unfamiliar concept especially in the mathematical sciences, it was only natural that this was a challenging experience with a rather steep learning curve. Notwithstanding the challenges, we found the outcomes of the project to be beneficial to the students, instructor, the university and the community.

The Good News

A former Vice-Chancellor for Academic Affairs and Dean of the University of Minnesota, Morris once said, "The math project on designing efficient snow-plow routes for the city is a poster child for service-learning projects." This project has a strong academic component as well as a service component.

From an academician's view point, we found that doing service-learning projects has benefits that cover our mantras of excelling in teaching, research and service. In terms of teaching, a hands-on and real-world application of the mathematical theory covered in class provides a motivational tool for the instructor. Being able to work on a modeling problem in which you have direct contact with all the raw data and the people using it, is definitely refreshing, albeit challenging; thus it complements working on relatively simple textbook problems. While the process of solving the problem is going on, we noted additional research questions such as "what model would fit with extra constraints, i.e. what if there must be sequential execution of different machines?" and "can we decompose the graph into simple cycles so that we are guaranteed to cover each block in two of the cycles?" The first question is a modeling one while the second is more of a basic research question in graph theory. Last but not least, this project clearly is of service or outreach value to the community we reside in.

From the students' perspectives (which were obtained via written student evaluations at the end of the course), in addition to the mathematical content learned, they felt a sense of gratification and empowerment, knowing that what they did actually made a difference to the community they lived in or will live in for at least four years. Students were able to get a taste of being in the role of applied mathematicians and as valuable members of a research team. In addition, students had first hand experience in dealing and communicating with non-mathematicians and non-academicians; they learned what questions to ask and how to identify useful information that is pertinent to their project. Indeed, a few years after their graduation, students gave us unsolicited comments on how they used this experience in their careers, where they often work on group projects and as professional consultants.

In terms of the benefits to the university and the community, service-learning projects like this one provide opportunities to bring together some of the needs/problems of the community and the resources from the university campus. For a small town, usually there is limited funding to hire external organizations to do extensive analyses or problem solving. And, at a public institution of higher learning, the chance to apply theoretical ideas beyond the walls of the classroom is invaluable, let alone applying it directly to the surrounding community. It was a win-win situation for both parties.

Last but not least, these projects encouraged future partnerships and collaborations with groups beyond the university. In fact, after the end of the SEAMS Grant for the three mathematics and statistics courses at the University of Minnesota, Morris, several other courses and projects with service-learning components were conducted successfully by faculty across the campus in conjunction with different organizations in the city.

The Challenges To Watch For

In this section, we will discuss a few challenges that we faced and how we dealt with these challenges while working on this service-learning project.

Time Constraints

Whether the duration of the term is a 10 week quarter or a 15 week semester, the length of time is quite restrictive from the students' point of view in terms of trying to learn the material and then applying the concepts to their specific projects. We took on the service-learning project with the understanding that it would not require us to compromise on the contents of the curriculum, but then we had to teach the students the theoretical concepts before they could apply them. Timing within the course was especially crucial for the group of students who worked on the efficient snow plow routes project; we did not finish covering the theory of optimal eulerization of a graph, optimal perfect matching, and shortest paths until about two thirds into the course.

This does not mean that students did not start on their projects until they learned the specific mathematical concepts; a great deal of preliminary work was done on the information given to the class. As we mentioned at the beginning of this paper, having a full-time coordinator who was a wonderful liaison between the class (students and instructor) and the community agency was invaluable. The coordinator was able to schedule numerous meetings outside class, and even brought the director of public works into two class sessions. A few meetings with representatives from the community agencies took place during the first third of the course, so students had a good "big picture" idea of their ultimate goal on their specific project.

Resources and Communications

Incorporating service-learning requires time and resources. Unless you (faculty) have a lot of time to spare, our advice is that you apply for some grants (external or internal) so that you can afford to have a person who coordinates laborious administrative tasks and paper work. In addition, it is very important to have some discretionary funds when student groups need to make a visit to the community agency or to go on a field trip that is essential to their project. It would have been impossible for us to implement service-learning had we not received such external funding. One good place to start searching for funding opportunities is Campus Compact, a national coalition of about 850 college and university presidents committed to the civic purposes of higher education.

Another challenge throughout all of these service-learning projects has been keeping up with all of the communications with community organizations. Delays are inevitable; there could be complex bureaucratic channels in an agency through which we might need to obtain some pertinent data or information. We were very fortunate to have an outstanding coordinator who had enough intimate knowledge of the community organizations to minimize delays and to promptly schedule meetings with the people concerned.

Process of Modeling

Not surprisingly, we found that pruning the questions given to us and fitting them into a mathematical context were daunting and time-consuming tasks. This issue is not restricted to working on community projects; this is an accepted occurrence among professional applied mathematicians who work on problems from industry. On the other hand, the process of going from plain questions to mathematical models is the fun part of being a practicing applied mathematician.

To minimize misunderstandings and complications from the questions posed by the community agencies, in the case of service-learning within a course with a specific starting time and a set length of time, our advice is for the instructor and coordinator to carefully plan their first organizational meeting with the students. In particular, after initial contacts and discussions with the community organizations, we advise the instructor to do some preliminary pruning of the questions before the course starts. The fact of the matter is that community organizations might have restrictions and deadlines that do not coincide neatly with any academic calendars.

Follow-ups

Even though the project was completed by the end of the term, we (instructor and coordinator) did some follow-ups with the community agencies. We strongly advise you to do something similar to this either for the purpose of closure or for the sake of future collaborations. In our case, we were invited to give presentations of our students' results at a regularly scheduled meeting of the city and its planning commission, so that the city had the opportunity to ask questions and make comments on any proposals that students made. Each of the community agencies was also given copies of the final reports written by the students. One reality we observed was that it takes time to implement changes, especially at state-run agencies. Nevertheless, changes do occur slowly but surely.

Closing Remarks

Although the project we discussed in this paper was done in a mathematics course, we would like to make it clear that the contents of discrete and combinatorial mathematics are very interdisciplinary. If you have a course in computer science or engineering that covers similar topics, and if you are proactively seeking service-learning projects, you can approach your community to see if this is applicable.

Overall, the integration of service-learning into discrete and combinatorial mathematics was arguably a success, notwithstanding the obstacles that the students encountered. The results and proposals recommended by the students were very thorough, meticulously thought out, and well-organized. This experience was the beginning of a collaborative partnership between the City of Morris and the University of Minnesota, Morris.

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Author Background

Peh Ng received a B.S. in Mathematics and Physics from Adrian College, Michigan; an M.S. in applied mathematics from Purdue University; and a Ph.D. in Operations Research and Combinatorial Optimization, also from Purdue University. She is currently a Morse-Alumni Distinguished Teaching Professor of Mathematics and an Associate Professor of Mathematics at the University of Minnesota, Morris. Her areas of research include operations research, discrete optimization and graph theory, within which she has several publications.